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Oscillations of Lake Wakatipu

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Abstract

RESULTS of experimental and theoretical investigations of the oscillations of the surface of Lake Wakatipu are described.

INTRODUCTION

IT has long been known that the waters of Lake Wakatipu, in the South Island, New Zealand, rise and fall with a slow rhythm; a behaviour which is only obvious on windless days, as it is otherwise masked by the surface waves. Beattie (1945) relates several Maori legends accounting for it, one ascribing it to the beating heart of a buried giant. But Lake Wakatipu is by no means unique in this property; Lake Wanaka also oscillates in the same way, as indeed will any closed body of water.

The fact that lakes rose and fell in regular motion seems to have been first noticed by the Swiss engineer, Duillier, in 1730, who recorded the level of Lake Geneva, but not until 1869 did we have any accurate knowledge of the phenomenon or a reasonable description of it. This understanding came from the extensive work carried out by Forel, also on Lake Geneva, which exhibits an oscillation of up to 5 feet, which is somewhat greater than that exhibited by most other lakes. Forel was able to show that the whole body of water was swaying from end to end (or side to side, or both), that is, as the level rose at one end, it fell at the other, as can be demonstrated quite simply in a teacup. The phenomenon is known as a " seiche ", a name common even in Duillier's time.

Chrystal (1905a) gives an extensive bibliography of earlier work on the seiches of lakes in Europe, along with an interesting historical account, while a more up-to-date list can be found in Hutchinson (1957). Bottomley (1955, 1956a and 1956b) conducted theoretical and experimental work on Lake Wakatipu, but little study of Lake Wakatipu has been made since then, although comments appear from time to time in the local press. This is perhaps because the magnitude of the oscillation of Lake Wakatipu tends to be somewhat larger than that of many other lakes.

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Experimental Observations

A study of the seiches in a lake calls, ideally, for simultaneous observations to be made, at a number of places, of the rise and fall of the water level. However, this requires either self-recording apparatus, or a team of observers. On the other hand, it is possible to investigate the seiche at any one particular place, with simple apparatus.

Bottomley (1955 and 1956b) showed that the fundamental oscillation (called a uninodal seiche) has a period of 52 minutes, and the first overtone (or binodal seiche) a period of 27 minutes, though superimposed on some of the recordings taken at Bob's Cove, was a much faster oscillation of period 4.28 minutes. He suggested that this could be interpreted as a transverse, rather than a longitudinal seiche.

Very rough observations by the author at Wilson's Bay with even simpler apparatus (based on the method described by Chrystal (1906; pp. 393–396)) confirm the period of the binodal seiche at about 27 minutes, the trace obtained being shown in Fig. 1. As the measurements were taken at a point which is very

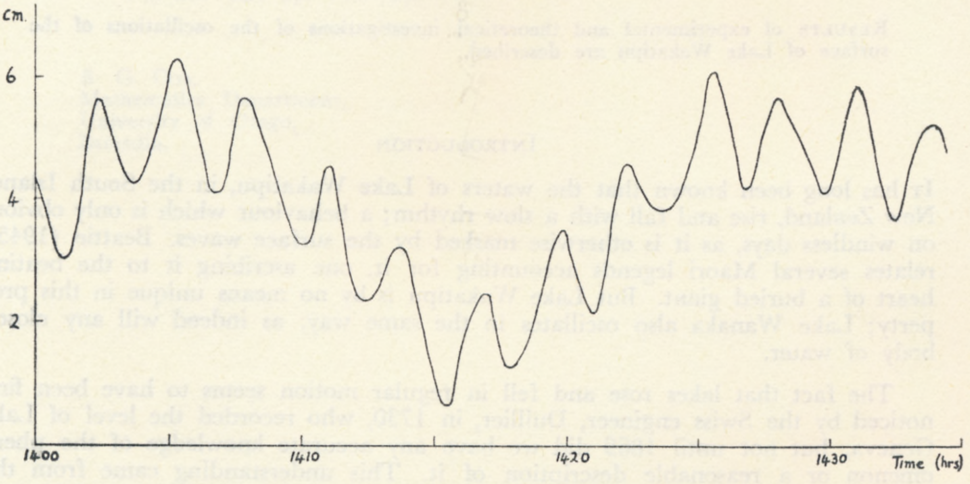

Figure 1.—Record of lake surface level taken at Wilson's Bay, Lake Wakatipu, on the afternoon of 27 October 1963, showing two distinct oscillations, one of period 27 minutes and range 6 cm, the other of period 2.9 minutes and range 2 cm.

near the position of the nodal line of the fundamental oscillation, no information could be expected about the latter. However, this time there was an oscillation of period 2.86 minutes superimposed on the record, and this is thought to be significantly different from the 4.28 minutes measured by Bottomley.

Residents at Walter Peak Station have frequently observed an oscillation with a period of about 5 minutes, but as it is not known whether these are accurately made recordings or merely estimates of the passage of time, little significance can be attached to them, except that they are unlikely to be connected with longitudinal oscillations of the whole lake. This is because the higher "harmonics" of a longitudinal seiche are damped out quickly, and so most unlikely to be observed by the naked eye. Chrystal (1908) found that a complex oscillation quickly damps to a composition of uninodal and binodal oscillations, called a dicrote seiche.

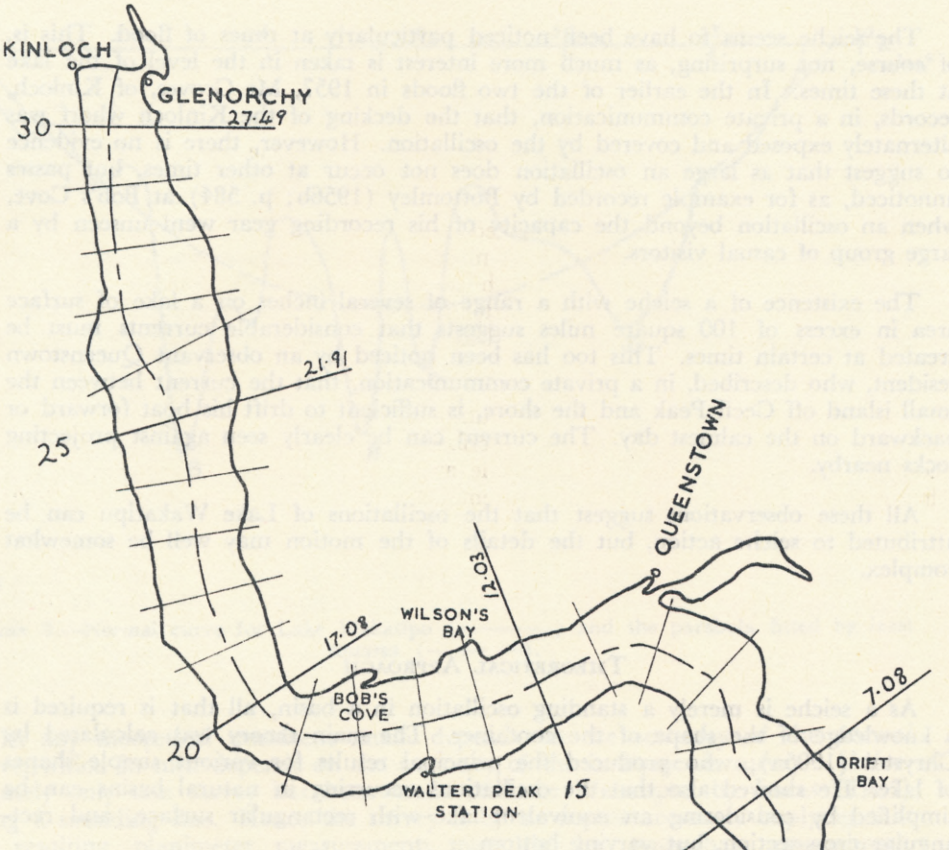

FIGURE 2.—Sketch map of Lake Wakatipu showing relevant geographical details, the line of greatest depth (——— ———), and the transversals used for calculating "the normal curve". Every fifth transversal is referenced; the number on the left being a reference number, and the number on the right (times 10^{11} sq. cm.) being the area of the surface of the lake between the transversal and the Kingston end of the lake (i.e., the quantity x in the text). Alongside transversal 5 is a sketch describing b(x) and h(x).

The seiche seems to have been noticed particularly at times of flood. This is, of course, not surprising, as much more interest is taken in the level of the lake at these times. In the earlier of the two floods in 1957, Mr Groves, of Kinloch, records, in a private communication, that the decking of the Kinloch wharf was alternately exposed and covered by the oscillation. However, there is no evidence to suggest that as large an oscillation does not occur at other times, but passes unnoticed, as for example recorded by Bottomley (1956b; p. 584) at Bob's Cove, when an oscillation beyond the capacity of his recording gear went unseen by a large group of casual visitors.

The existence of a seiche with a range of several inches on a lake of surface area in excess of 100 square miles suggests that considerable currents must be created at certain times. This too has been noticed by an observant Queenstown resident, who described, in a private communication, that the current between the small island off Cecil Peak and the shore, is sufficient to drift his boat forward or backward on the calmest day. The current can be clearly seen against projecting rocks nearby.

All these observations suggest that the oscillations of Lake Wakatipu can be attributed to seiche action, but the details of the motion may well be somewhat complex.

Theoretical Approach

As a seiche is merely a standing oscillation in a basin, all that is required is a knowledge of the shape of the container. The main theory was calculated by Chrystal (1905a), who produced the principal results for various simple shapes of lake. He showed also that the oscillations occurring in natural basins can be simplified by considering an equivalent lake with rectangular surface, and rectangular cross-section, but varying bottom.

The procedure is as follows. The line of greatest depth is traced on a bathymetric survey map of the given lake. Any number of transversals perpendicular to this line are taken, and for each transversal, the surface area from one end of the lake to the transversal is measured (usually by planimeter). This quantity is denoted by x. The area of the vertical cross-section through each transversal $h(x)$ is found (also by planimeter), and multiplied by $b(x)$ the length of the transversal lying in the surface of the lake, as shown in Fig. 2. The product $b(x)h(x)$, denoted by $D(x)$, is then plotted against x, producing what Chrystal called " the normal curve ". The normal curve for Lake Wakatipu is shown in Fig. 3. It was shown by Chrystal that a lake with rectangular surface, and bottom corresponding to the normal curve has the same hydrodynamical properties as the original complex shape provided the breadth of the lake is suitably scaled to give the same surface area as the original. Longitudinal oscillations are then considered in this " equivalent lake ".

Bottomley's method (1956a) combined experiment and theory. He made a scale model of the equivalent lake, filled it with water, and created standing oscillations in the model. The observed periods of the model were then scaled to predict the periods occurring in the lake itself, obtaining 50.9 minutes and 27.7 minutes as the suggested periods for the first two oscillations. Unfortunately a numerical error in the scaling factor went unnoticed, and his predicted periods should be 57.2 minutes and 31.1 minutes, respectively, and hence the agreement between experiment (52 and 27 minutes) and theory is not as good as at first sight.

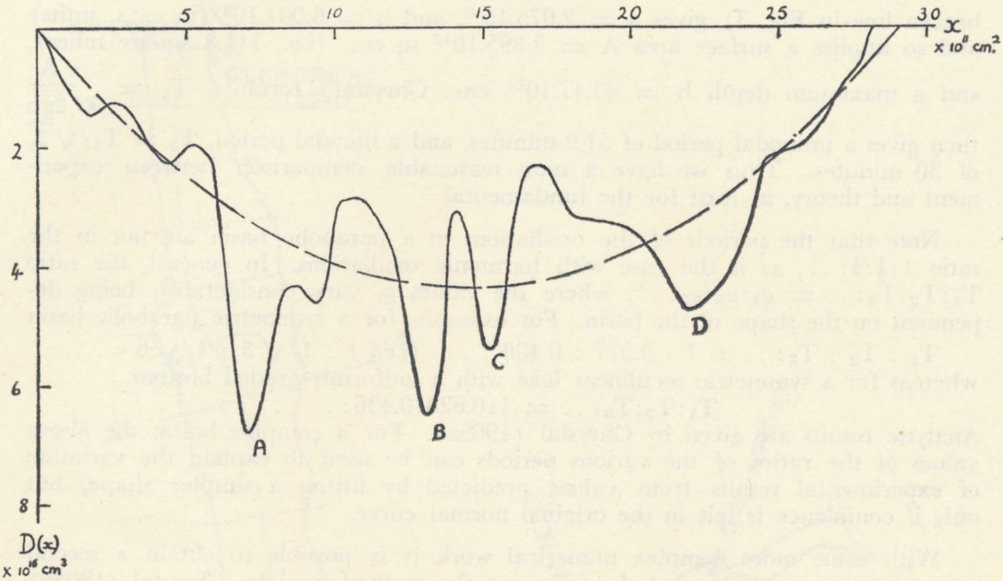

FIGURE 3.—Normal curve for Lake Wakatipu (———) and the parabola fitted by least squares (— — —).

In any theoretical treatment much depends on the accuracy of the normal curve, which in turn depends on the accuracy of the bathymetric survey. Unfortunately only one survey has been made of Lake Wakatipu; by Lucas (1904) using a sounding line. Hence the survey is by no means accurate, implying that the resulting planimeter measurements may also be somewhat approximate. A minor error in the placing of the sides of the glacial trough at any cross-section in a lake as deep as Lake Wakatipu results in a considerable error in h(x), and hence in D(x). The scale of 1 to 200,000 of Lucas's map results in further inaccuracy. Hence it is heartening to find that the above normal curve is substantially the same as that calculated by Bottomley (1956a; p. 52).

It is difficult to know how much significance should be attached to the deep valleys A, B, C and D in the curve shown in Fig. 3. Each one corresponds to a major bay (e.g., Drift Bay, Wilson's Bay) where inaccuracies in the survey are considerable, and may be in either direction. Hence it is not possible at present, or at least until a more accurate survey has been made, to attempt a sophisticated analysis, and only approximate theories have any hope of success.

Thus, as a first approximation, it is reasonable to ignore the peaks and valleys, and to fit a simple curve as closely as possible to the given data for the normal curve. The seiche periods can then be calculated by inserting numerical values in the formulae developed by Chrystal (1905a; p. 622). This should give a reasonable result for the period of the fundamental, but cannot be expected to predict the periods of higher order oscillations with any degree of accuracy, as the influence of irregularities will be much greater in these cases (see Chrystal (1905a)).

The first obvious curve to fit (by least squares) is a symmetric parabola of the form $D(x) = -ax^2 + bx$; i.e., not necessarily assuming that the parabola passes through both end points of the normal curve. This was necessary as the above normal curve implies a surface area of 108.2 square miles, whereas Lucas (1904) gives an area of 112.3 square miles. Fitting such a curve (shown by the broken line in Fig. 3) gives $a = 2.075.10^{-9}$, and $b = 6.007.10^{3}$ (in c.g.s. units) and so implies a surface area $A = 2.895.10^{12}$ sq. cm. (i.e., 111.8 square miles), and a maximum depth $h = 43.47.10^{14}$ cm. Chrystal's formula, $T_1 = \frac{\pi A}{\sqrt{2gh}}$ then gives a uninodal period of 51.9 minutes, and a binodal period, $T_2 = T_1/\sqrt{3}$, of 30 minutes. Thus we have a most reasonable comparison between experiment and theory, at least for the fundamental.

Note that the periods of the oscillations in a parabolic basin are not in the ratio $1:\frac{1}{2}:\frac{1}{3}:..$, as is the case with harmonic oscillations. In general, the ratio $T_1:T_2:T_3:.. = g_1:g_2:g_3:..$, where the values g_i vary considerably, being dependent on the shape of the basin. For example, for a symmetric parabolic basin

$$T_1 : T_2 : T_3 : .. = 1 : 0.577 : 0.408 : ... \text{ (i.e., } 1 : 1/\sqrt{3} : 1/\sqrt{6} : .\text{)},$$

whereas for a symmetric rectilinear lake with a uniformly graded bottom,

$$T_1:T_2:T_3:.. = 1:0.628:0.436:.. \quad .$$

Analytic results are given by Chrystal (1905a). For a complex basin, the above values of the ratios of the various periods can be used to explain the variation of experimental results from values predicted by fitting a simpler shape, but only if confidence is felt in the original normal curve.

With some more complex numerical work it is possible to obtain a second approximation to the periods by following the method used by Chrystal (1905b) for the estimation of the periods of Lochs Earn and Treig. This method fits a pair of parabolas, one on each side of the deepest point, thus avoiding the assumption of the symmetric parabola method that the deepest point of the equivalent lake is at the centre of the lake, and the resulting periods are used as approximations to a numerical solution of an equation for the periods. However, this method requires some accuracy in the initial bathymetric survey, so it is of doubtful value at the present time, particularly as the position of the deepest point is so uncertain. For example, with reference to Fig. 3, if valley A is taken as representing the deepest point, the approximation by fitting a pair of parabolas gives the fundamental period as 51.2 minutes, whereas if valley B is chosen, which is probably more realistic, the first approximation is 50.1 minutes. In either case there is little difference from the value predicted by the symmetric parabola method (51.9 minutes), and for the reasons given above, predictions of the periods of higher order oscillations, which can be obtained by the same method, are unlikely to be reliable. Hence at present there seems little point in proceeding with more sophisticated numerical techniques which take into account the effects of the peaks and valleys in the normal curve, until a more accurate bathymetric survey is available.

Other Oscillations

As mentioned earlier, observers have noticed oscillations with periods 2.86 minutes and 4.28 minutes at Wilson's Bay and Bob's Cove respectively. It appears likely that the former is a transverse seiche in the middle arm of the lake. A rough approximation to <u>any</u> fundamental seiche can be found by "Merian's Rule"—i.e., $T_1 = 2l/\sqrt{gh}$, for the seiche in a rectangular basin of length l and depth h (see Hutchinson, p. 301). Taking the appropriate data for a transverse section at Wilson's Bay, l = 3 miles, h = 1,150ft, the above rule gives the period of the fundamental as 2.8 minutes. As the shape of the glacial trough closely resembles a rectangular basin at this point, it is thought that the agreement between 2.86 and 2.8 minutes is strong evidence in support of the existence of a transverse seiche.

Where then does the observation of 4.28 minutes at Bob's Cove fit in? Merian's Rule here gives a period of approximately 3 minutes. It is not unlikely that the influence of the corner at White Point is creating a complex oscillation in the middle arm composed of harmonics of both longitudinal and transverse seiches. It is probably purely coincidence that a period of 4.28 minutes corresponds quite closely to the period of a binodal seiche in the middle arm alone, as an approximate calculation by Merian's Rule will confirm. Such a seiche would have a node quite close to Bob's Cove, and thus an oscillation of small or even negligible amplitude would be expected at this place. Bottomley's observation of a large amplitude, which he showed does not correspond to an oscillation of the cove alone, seems to suggest that the above hypothesis of a binodal seiche is also invalid.

The true nature of these recorded observations at Bob's Cove, at Wilson's Bay, and at Walter Peak Station can only be determined by recordings taken at two or more places for a reasonable length of time.

Theories of Cause

Both Chrystal and Hutchinson suggest some of the possible causes generating seiches, and Chrystal (1908) was able to confirm by his work on Loch Earn that certain variations in barometric pressure, and other factors were followed by the formation of a seiche.

Considering the regular, and frequent, changes in meteorological conditions in New Zealand, it would be interesting to see if any major factor is the cause of a seiche on Lake Wakatipu; for example, the regular variation from North-West to South-West weather. It is unlikely to be easy to distinguish between the effect of differences in barometric pressure at two points of the lake, and the effect of cessation of a strong wind—both likely causes—but it would be interesting to try.

Another interesting conjecture is the possible influence of the daily variation in the volume of water flowing from the snow fed rivers entering the lake. Could this daily variation trigger a seiche of period less than an hour?

A tentative theory of the cycle of events is as follows. The initial disturbance, whatever it may be (wind, periodic pressure variation, heavy rain, or some such), creates a complex longitudinal oscillation with several overtones. This quickly damps out to an oscillation consisting mainly of uninodal and binodal components —i.e., a dicrote seiche. The longitudinal oscillation in turn creates a transverse seiche in the shorter middle section of the lake. The resulting oscillations may persist for some days, until a further disturbance occurs, which destroys the existing seiche and creates a new one, or possibly reinforces the fading seiche. The cycle then repeats itself. It is not unrealistic to suppose that Lake Wakatipu oscillates for a large proportion of the year, since a seiche has been observed whenever anyone has attempted to measure the level of the surface of the lake.

Acknowledgments

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