# Elastic Constants of the Earth's Mantle.

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THE solid portion of the Earth outside the central core is commonly referred ·to in seismological work as the Earth's "mantle." It extends to a depth below the surface which Jeffreys (1939) has now fixed as 2900 km. within an error of not more than a few kilometres. The purpose of the present paper is to obtain figures giving the usual elastic constants for the materials of the Earth's mantle, apart from the crustal layers. Values for the crustal layers have been ·obtained from near earthquake studies and are liable to vary somewhat from region to region. Studies of distant earthquakes show that, below the crustal layers, the Earth conforms closely to spherical symmetry, and it is now possible to give fairly accurate values of the elastic constants at various depths below these layers.

The determination to fair accuracy becomes possible on account of the recent accurate determination by Jeffreys (1939A) of the velocities $\alpha$ and $\beta$, respectively, of P and S seismic waves throughout the Earth's mantle, and also the work of the writer on the density distribution. In the following calculations, values of the density $\rho$ will be those obtained in the most recent publication (Bullen, 1940) of the writer on the Earth's density variation, and use will be made of the assessment there given of the limits of error in the density figures.

The Lamé elastic constants $\lambda$ and $\mu$ may thus be found directly from the standard equations:—

$$\alpha^2 = (\lambda + 2\mu)/\rho ,$$
$$\beta^2 = \mu/\rho$$

·The bulk modulus or incompressibility $k$ and Poisson's ratio $\sigma$ are ·obtained using:—

$$k = \lambda + \tfrac{2}{3}\mu ,$$
$$\sigma = \tfrac{1}{2}\lambda/(\lambda+\mu) .$$

In Table I the results are expressed in terms of the depth $d$ km. below the Earth's surface, the thickness of the crustal layers being ·taken as 33 km. The units of $\lambda$, $\mu$ and $k$ are $10^{-12}$ dynes/cm. in each ·case; $\sigma$ of course is a pure ratio.

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For theoretical purposes, it is sometimes convenient to have various results in terms of values of $r/R$, where $R$ is the radius of the Earth's sub-crustal material, and $r$ the distance of a point from the Earth's centre. For this purpose Table II has been constructed. Values of the density $\rho$, in gm./cm.³, as found during the preparation of the density paper (Bullen, 1940), are also given in Table II to three decimal places. The third place has been included merely to facilitate theoretical work and has no significance in practice.

The density figures used were shown (1940) to be probably accurate within rather less than 1 per cent. at practically all depths, and it is likely that, except between the depths of 413 and 984 km. (corresponding to values of $r/R$ of 0·94 and 0·85), the values of the elastic constants given are accurate within an order of 1 per cent. Between the particular depths just mentioned, the uncertainty may be a little greater on account of a slightly increased uncertainty in the velocity figures in the region here involved.

Values of the constants $\mu$, $k$ and $\sigma$ to two significant figures are also given by Gutenberg and Richter (1939), using a completely independent set of velocity figures, and earlier density figures of the present writer. Within the errors of the two determinations, the agreement is very satisfactory.

As with the density and velocities, there are sudden changes in the gradients of the elastic constants when $r/R$ is 0·94, i.e. at a depth of about 413 km.

<center>TABLE I.</center>

| $d$ | $\lambda$ | $\mu$ | $k$ | $\sigma$ |
|---|---|---|---|---|
| 33 | 0.74 | 0.63 | 1.16 | 0.269 |
| 100 | 0.80 | 0.67 | 1.24 | 0.272 |
| 200 | 0.90 | 0.74 | 1.38 | 0.275 |
| 300 | 1.01 | 0.81 | 1.54 | 0.277 |
| 400 | 1.12 | 0.89 | 1.71 | 0.279 |
| 413 | 1.14 | 0.90 | 1.73 | 0.280 |
| 500 | 1.42 | 1.10 | 2.15 | 0.283 |
| 600 | 1.69 | 1.32 | 2.57 | 0.282 |
| 700 | 1.90 | 1.52 | 2.91 | 0.278 |
| 800 | 2.06 | 1.69 | 3.19 | 0.275 |
| 900 | 2.20 | 1.81 | 3.40 | 0.275 |
| 1000 | 2.33 | 1.89 | 3.59 | 0.276 |
| 1200 | 2.54 | 2.03 | 3.89 | 0.278 |
| 1400 | 2.76 | 2.15 | 4.20 | 0.281 |
| 1600 | 3.00 | 2.27 | 4.52 | 0.284 |
| 1800 | 3.27 | 2.39 | 4.87 | 0.288 |
| 2000 | 3.55 | 2.51 | 5.23 | 0.292 |
| 2200 | 3.81 | 2.63 | 5.57 | 0.295 |
| 2400 | 4.06 | 2,76 | 5.90 | 0.298 |
| 2600 | 4.32 | 2.88 | 6.23 | 0.300 |
| 2800 | 4.47 | 3.01 | 6.47 | 0.299 |
| 2900 | 4.49 | 3.03 | 6.51 | 0.299 |

TABLE II.

| d | r/R | ρ | λ | μ | k | σ |
|---|---|---|---|---|---|---|
| 33 | 1.00 | 3.320 | 0.74 | 0.63 | 1.16 | 0.269 |
| 96 | 0.99 | 3.379 | 0.80 | 0.67 | 1.24 | 0.272 |
| 160 | 0.98 | 3.436 | 0.86 | 0.71 | 1.33 | 0.274 |
| 223 | 0.97 | 3.491 | 0.92 | 0.75 | 1.42 | 0.276 |
| 286 | 0.96 | 3.543 | 0.99 | 0.80 | 1.52 | 0.277 |
| 350 | 0.95 | 3.594 | 1.06 | 0.85 | 1.62 | 0.278 |
| 413 | 0.94 | 3.643 | 1.14 | 0.90 | 1.73 | 0.280 |
| 476 | 0.93 | 3.824 | 1.35 | 1.05 | 2.04 | 0.283 |
| 540 | 0.92 | 3.988 | 1.54 | 1.19 | 2.33 | 0.283 |
| 603 | 0.91 | 4.136 | 1.70 | 1.33 | 2.58 | 0.282 |
| 666 | 0.90 | 4.267 | 1.84 | 1.46 | 2.81 | 0.279 |
| 731 | 0.89 | 4.383 | 1.95 | 1.58 | 3.00 | 0.277 |
| 794 | 0.88 | 4.480 | 2.05 | 1.68 | 3.17 | 0.275 |
| 921 | 0.86 | 4.624 | 2.23 | 1.83 | 3.45 | 0.275 |
| 1047 | 0.84 | 4.708 | 2.38 | 1.92 | 3.66 | 0.277 |
| 1174 | 0.82 | 4.785 | 2.51 | 2.01 | 3.85 | 0.278 |
| 1301 | 0.80 | 4.859 | 2.64 | 2.09 | 4.04 | 0.279 |
| 1427 | 0.78 | 4.931 | 2.79 | 2.17 | 4.24 | 0.281 |
| 1554 | 0.76 | 5.002 | 2.94 | 2.25 | 4.44 | 0.283 |
| 1681 | 0.74 | 5.070 | 3.10 | 2.32 | 4.66 | 0.285 |
| 1807 | 0.72 | 5.137 | 3.28 | 2.40 | 4.88 | 0.288 |
| 1934 | 0.70 | 5.204 | 3.46 | 2.47 | 5.11 | 0.291 |
| 2061 | 0.68 | 5.269 | 3.63 | 2.55 | 5.33 | 0.293 |
| 2188 | 0.66 | 5.333 | 3.79 | 2.62 | 5.54 | 0.295 |
| 2315 | 0.64 | 5.396 | 3.95 | 2.70 | 5.75 | 0.297 |
| 2442 | 0.62 | 5.459 | 4.12 | 2.78 | 5.97 | 0.299 |
| 2568 | 0.60 | 5.521 | 4.28 | 2.86 | 6.18 | 0.300 |
| 2695 | 0.58 | 5.583 | 4.44 | 2.94 | 6.39 | 0.301 |
| 2822 | 0.56 | 5.645 | 4.48 | 3.02 | 6.49 | 0.299 |
| 2885 | 0.55 | 5.676 | 4.49 | 3.03 | 6.51 | 0.299 |

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