The Variation of Gravity Within the Earth.

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In two recent papers, Benfield (1937) and Olczak (1938) have given figures for the attraction due to gravity at various points in the interior of the earth, the calculations being based on the writer's solution (1936) of the variation of density within the earth.

As in the latter paper, let ρ be the density at a point P distant r from the earth's centre, and M the mass enclosed by a concentric sphere through P. Let α, β be the velocities of P and S seismic waves at P, and γ the constant of gravitation. Then in the course of the writer's solution of the density problem, the following equation had been set up:—

$$\frac{d\rho}{dr} = -\gamma M r^{-2} \rho / (\alpha^2 - \tfrac{4}{3}\beta^2),$$

and had been applied successively to give the rate of density alteration at each point taken in the earth's interior. The right-hand side contains the factor $\gamma M r^{-2}$ which had been determined explicitly on each occasion; and this expression of course gives directly the gravitational attraction at distance r from the earth's centre.

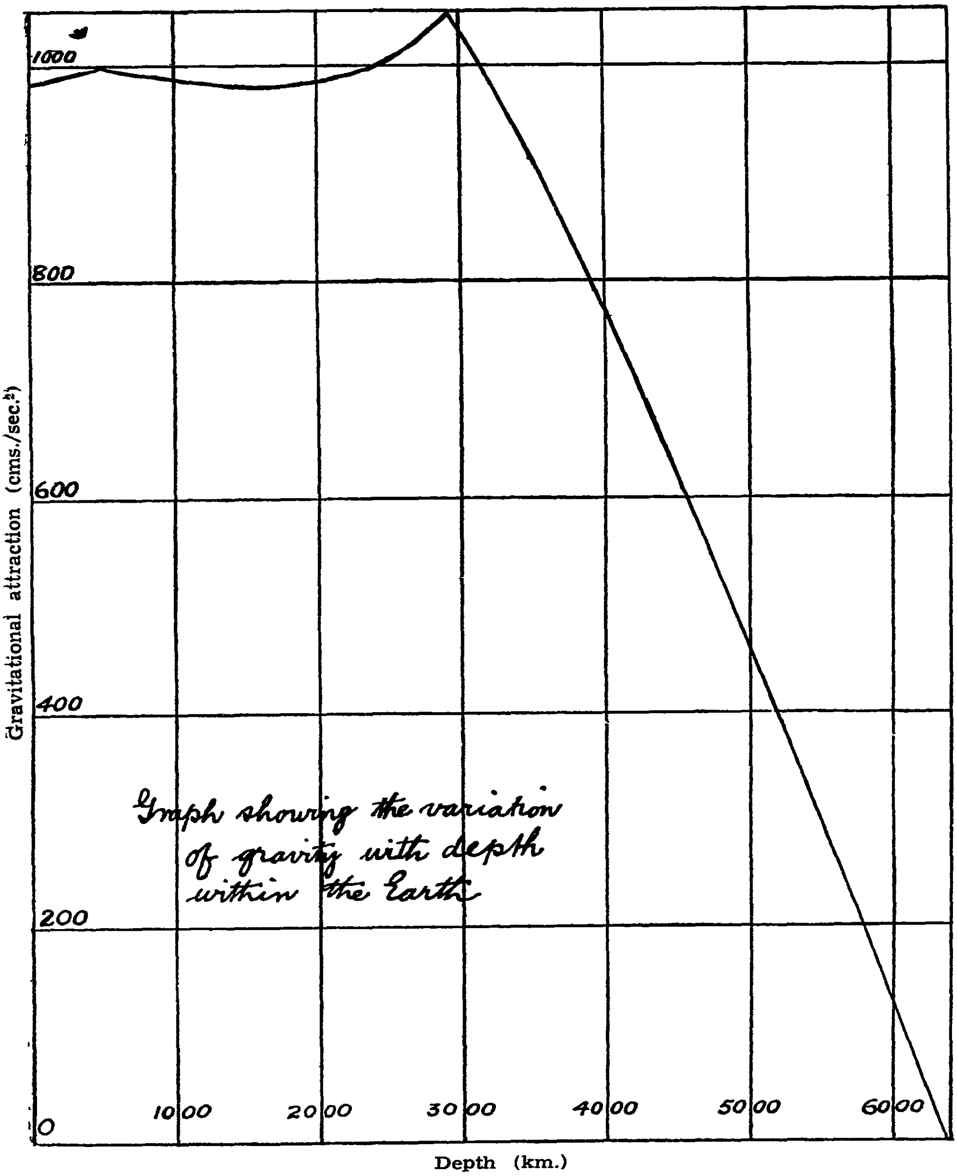
The results of Benfield and Olczak were obtained by using the final published density figures of the writer, and accordingly involved a fair amount of additional computation. It may therefore be of interest to give the actual values of the function $\gamma M r^{-2}$ as found directly by the writer, such values being subject to the minimum of computation errors.

The values to be given are based on the auxiliary material prepared during the writer's revised solution (1937) of the density variation. Allowance has further been made for the improved estimate of the earth's mass at $5 \cdot 976 \times 10^{27}$ gms., as given by Olczak. In this connection it is of interest to point out that the same figure for the earth's mass was independently given recently by Jeffreys (1937). A corresponding approximate adjustment for this was made by Benfield and Olczak, but more accurate allowance is possible using the auxiliary figures of the writer's earlier papers.

The values thus obtained by the writer for the gravitational attraction g cms./sec.^{2} at various depths d km. below the earth's surface are as follows—

d	*g*	*d*	*g*	*d*	*g*
0	982	2000	985	4200	714
100	986	2200	990	4400	654
200	989	2400	1000	4600	592
300	992	2600	1014	4800	528
400	995	2800	1034	5000	463
474	997	2920	1048	5200	398
600	995	3000	1031	5400	331
800	991	3200	986	5600	264

d	g	d	g	d	g
1000	988	3400	936	5800	196
1200	985	3600	885	6000	127
1400	982	3800	831	6200	59
1600	981	4000	774	6371	0
1800	982				



The general variation of g is exhibited in the accompanying diagram, and is similar to the results of Benfield and Olczak, although the values are for the most part a little higher. This is due partly to the use of the writer's revised solution (1937) for the density variation, and partly to the more accurate adjustment corresponding to the increased mass adopted for the earth; the effect of the latter is to increase the density more at the greater depths than near the earth's surface. The discontinuities of gradient at depths of 474 and 2920 km. are associated with maxima of 997 and 1048 cms./sec^2 respectively. The minimum value of g in the earth's outer shell is about 981 cms./sec.^{2}, and occurs at a depth of about 1620 km. Inside the central core gravity diminishes to zero by a nearly linear law.

The adoption of an increased mass affects slightly the density figures given by the writer (1937) for the central core. These now range from 9·77 at the boundary to 12·29 at the earth's centre. Above the central core there is virtually no alteration necessary.

A few further remarks are necessary concerning the solution given. First, it is not yet certain from the seismological evidence exactly how sharp is the new discontinuity placed at 474 km. depth. If the transition is fairly gradual, the effect will be essentially to smooth out the discontinuity in the gradient of the gravity curve at this depth, but other features of the curve will be preserved. Any modification made will be fairly small, since, as pointed out by the writer (1938) there are a number of independent lines of evidence which all support the occurrence of a change in composition in the earth at a depth of order 500 km.

Secondly, Gutenberg and Richter (1938) have recently made the suggestion that the central core itself may be composite with possibly two changes of material inside. This is liable to affect slightly the variation of gravity within the central core, but the general solution for the whole earth should not be greatly disturbed, since the mass of the central core is less than a third of the mass of the earth.

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