relative dimensions, and the most advantageous location of the plates, as well as those relating to the connecting machinery, and the most economical application of the power employed. These, however, are but questions of constructive detail which do not affect the object of this paper, which is merely to establish the principle involved.

It is, of course, only by actual trial that the practicability of the idea could be demonstrated. Still, the advantages of such a contrivance, were it capable of being carried into effect, are so obvious that it would be well worth while to make a series of experiments, in the first instance with a vessel of small dimensions, of which the cost would be comparatively trifling considering the interest at stake. And, though we may never expect that the action of the brake-fins on a ship will equal that of a Westinghouse brake on an express train, or even that of the oars on a skilfully-handled boat, still, if they will shorten by a cable's length the distance at which she will bring up, or reduce to any considerable degree the angle at which she will come round, they may be the means of giving many a "man overboard" a chance for his life, and help to minimise the increasing chances of one of the most appalling of disasters, a collision at sea.

Art. LXV.—*Mill on Demonstration and Necessary Truth.*

By William Carlile, M.A.

[*Read before the Wellington Philosophical Society, 8th July, 1891.*]

If any one should endeavour to ascertain what is the received doctrine in England at present with regard to the basis of mathematical demonstration, the true nature of the definitions of Euclid, and the ultimate evidence for the axioms, he would find himself met by a very remarkable diversity of opinion on the part of those who have been recognised as the highest authorities on psychology and metaphysics during the past half-century. To find anything like consistency, indeed, he would have to go back to the philosophers of the pre-Kantian age.

Hume's opinion was that the truths of pure mathematics were to be put into one class along with identical propositions, and that truths of matters of fact were to be put into another and altogether different class. However certain the latter might be, their certainty, in his view, depended upon an entirely different sort of evidence from that on which the certainty of the former depended. This doctrine, at any rate, afforded him a basis of classification which enabled him to avoid the hair-splittings and inconsistencies of the modern Humist school. The great leader of that school, Mr. J. S. Mill, takes all the axioms out of the class of necessary truths, and puts them into the class of truths of experience; and he thus, as every one is aware, arrives at the amazing opinion that we are not justified in asserting that two and two could never in any possible circumstances make five. His most distinguished disciple, Professor Bain, does not altogether follow him here. He, indeed, takes the axiom, "Things which are equal to the same thing are equal to one another," with its corollaries, out of the identical or implicated class, and puts it into the matter-of-fact or experience class; but he leaves others of them, such as the predication in regard to two straight lines that they cannot enclose a space, in the identical class. Mr. Mansel, who follows Kant, and uses Kantian phraseology, also puts some of the axioms in one class and some in another; but the strange thing is that the very axioms which Mr. Bain takes out of the identical class Mr. Mansel retains within it, and others which Mr. Bain thinks fall within it he leaves outside it. Professor Huxley, following Bain, says of the axiom, "Things that are equal to the same thing are equal to one another," that it is only a particular case of the predication of similarity—that is to say, I suppose, that it is a proposition of precisely similar import to this: "John is very like Thomas." It must be said, however, that Professor Huxley's views altogether on necessary truth and cognate questions very plainly betray the amateur. He remarks in a previous chapter of his treatise* that the certain reminiscence, "I was in pain yesterday," may be properly said to be necessary. If that were so there would be no distinction whatever between truths of demonstration and facts of memory; and in that case a very great part of all that Plato, Descartes, Kant, Locke, and even Hume himself have written would be words to which no meaning could be attached. Professor Huxley, however, only puts in the crudest form what are in truth the doctrines of his school, and what are professed as such, though in more guarded fashion, by its acutest thinkers. Mr. Spencer, for instance, if he would not call the statement, "I was in pain yesterday," a necessary truth, would so denominate the statement, "I am in pain now." Such statements may possess a degree of certainty that cannot be exaggerated; yet a very little reflection is sufficient to show that the evidence on which they rest is in no

* "Hume," in the English Men of Letters Series.

respect analogous to that which we have for the truth of the axioms of mathematics.

Of necessary truths, properly speaking, the one unfailing criterion is that they should always be truths in regard to abstractions—never, in any case, truths in regard to concrete realities. That the moon which I see above me is one, not two, cannot be a necessary truth, no matter how certain I am of it. I might, indeed, be under a not uncommon hallucination in affirming its unity; but the proposition that "one is not two" stands on altogether a different footing. One is not two. Why? Because it is by hypothesis one. It is this intrusion of an assumption which can be contradicted which makes possible a necessary truth, resting, as all such truths do, on the law of contradiction. As, however, the current confusion of thought with regard to abstraction is at the root of much of the confusion of thought in regard to necessary truth, it may be well at this point to endeavour to arrive at a sound opinion with regard to the true nature of this important mental process.

In a paper read before this Society about two years ago on "Professor Huxley's Metaphysics," I endeavoured to draw attention to the crudity of the opinions in regard to the formation of abstract conceptions expressed by that very Philistine representative of English empiricism. With your permission I will recapitulate a portion of what I then said: "This mental operation" [abstraction], Professor Huxley says, "may be rendered comprehensible by considering what takes place in the formation of compound photographs—when the images of the faces of six sitters, for example, are each received on the same photographic plate for a sixth of the time requisite to take one portrait. The final result is that all those points in which the six faces agree are brought out strongly, while all those in which they differ are left vague; and thus what may be termed a genuine portrait of the six, in contradistinction to a specific portrait of any one, is produced." Similarly he thinks, "In dreams one sees houses, trees, and other objects, which are perfectly recognisable as such, but which remind one of the actual objects as seen out of 'the corner of the eye,' or the pictures thrown by a badly-focussed magic-lantern. A man addresses us who is like a figure seen by twilight, or we travel through countries where every feature of the scenery is vague, the outlines of the hills are ill marked, and the rivers have no defined banks. They are, in short, *generic* ideas of many past impressions of men, hills, and rivers." Here it is plain enough that what is vague is confounded with what is generic. One might as well say that those of Turner's pictures which are successful in conveying the effect of a hazy atmosphere are generic as that a man seen in dreams like a figure by twilight is generic. The fact is that the abstraction "man" must cover both men dimly perceptible and men palpably obtruded in broad sunlight under our very eyes. It must comprise contradictions; hence it is that no image can possibly be made of it. It may not be without interest to turn to a discussion on the same subject about two hundred years old. In treating of the formation of general ideas Locke says, "For example, does it not require some pains and skill to form the general idea of a triangle (which is yet none of the most abstract, comprehensive, or difficult)? for it must neither be oblique nor rectangle, neither equilateral, equicrural, nor scalenon, but all and none of these at once." Bishop Berkeley, in his gravely sarcastic fashion, takes him to task over this description of the general idea of a triangle. "If any man," says he, "has the faculty of framing in his mind such an idea of a triangle as is here described, it is vain to dispute him out of it, nor would I go about it. All I desire is that the reader would certainly inform himself whether he has such an idea or not. And this, methinks, can be no hard task for any one to perform. What more easy than for any one to look a little into his own thoughts and then try whether he has, or can attain to have, an idea that shall correspond with the description that is here given of the general idea of a triangle—*neither equilateral, equicrural, nor scalenon, but all and none of these at once!*" Locke might possibly have answered that he did not mean by an idea precisely what we mean by a mental image; but this answer cannot be put forward on behalf of Professor Huxley with his compound-photograph and dream-representation theories. The difficulty did not escape Kant.* "No image," he observes, "could ever be adequate to our conception of a triangle in general. For the generalness of the conception it never could attain to, as this includes under itself all triangles, whether right-angled, acute-angled, &c., while the image would always be limited to a single part of this sphere." Kant is of opinion, therefore, that it is not images, but what he calls schemata, that lie at the foundation of general conceptions, and his theory, I think, accords with the facts of consciousness. At the same time it must be observed that there is no doubt that when we think of "triangle," "man," or "river" in the abstract the image of some individual triangle, man, or river passes before our mind. We know, when we consider the matter, that this image does not cover the contents of the general conception. We use it merely as a specimen. At the same time we can very readily

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* "Critique of Pure Reason:" "Of the Schemata of the Categories."

glide into fallacious thinking by forgetting that it is this and nothing more.*

Professor Bain thinks that in forming general conceptions we can do one of two things: (1) we may call up an image that embraces all the attributes of rivers in general, or (2) we may call up the image of any particular river and use it as a symbol by which to think of rivers in general. It is plain enough that it is only the last of these two things that any one can do. The concept "river," like every abstract concept, is a schema or definition, not an image. It rests on a judgment or a series of judgments. In the simplest possible case—say, that of the concept "one" or the concept "blue"—there is, at any rate, behind such concepts the judgment that the name "one" or the name "blue" shall apply to the number "one" and the colour "blue" exclusively, and not to any other colour or number. This judgment is what Professor Huxley somewhere calls "the convention that underlies intelligible speech." He might have added, "that underlies rational thought." It is this judgment that is contradicted when we say, "One is two," or "Blue is green." Behind the mere sensation caused by one object, or by a blue object, there is, of course, no judgment to contradict. And it appears to be the easiest thing possible for even the very acutest thinkers to confound, in this respect, the sensation with the concept. To deny "that blue is not green," Mr. Mill says, "involves no logical contradiction. We could believe that a green thing may be blue as easily as we believe that a round thing may be blue if experience did not teach us the incompatibility of the former attributes and the compatibility of the latter."† This is all based upon the assumption that to affirm, "Green is not blue," is equivalent to affirming, "This green thing before me is not a blue thing;" while what it is really equivalent to is, "If this thing is green it is not blue." It is "a proposition concerning a proposition, the subject of the assertion being itself an assertion."‡ "Two straight lines

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* Mr. Mill, for instance, as will be seen further down, affirms that the reason why, in his view, we can gain from experience what seems to be axiomatic certainty in regard to geometrical truths, is to be found in the capacity of geometrical forms for being painted in the imagination with a distinctness equal to reality. We can thus, he says, copy lines and figures, and argue from the copies as we would from the realities. Granted that we can copy them as well in imagination as on a blackboard, what we argue from is not the specimen on the board, but the rule in accordance with which, or perhaps only in a rough approximation to which, it is drawn. This fallacy of confounding the functions of the specimen with those of the schema has been a fruitful source of error.

† "Examination of Sir W. Hamilton's Philosophy," 4th ed., p. 486.

‡ Mill's definition of a conditional proposition. "Logic," People's Ed., p. 53.

cannot enclose a space" is not equivalent to saying, "These two lines which I judge to be straight cannot enclose a space." That, so far from being necessary, would probably not even be true. What it is equivalent to is plainly this: "If these two lines are straight they cannot enclose a space." Mr. Mill shows himself occasionally to be aware of the existence of such assumptions behind every general conception. He tells us, for instance, that in naming* "we create an artificial association between attributes and a certain combination of articulate sounds." This means that for the purposes of thought and intercourse we agree that the particular colour we know as green shall have the name "green." It is plain, then, that if we afterwards proceed to say that the name "green" is equally applicable to some other colour, such as blue, we break our convention, we sublate our hypothesis, and we involve ourselves in as unmistakable a logical contradiction as it is possible to conceive.

Similarly, if we are asked, "On what sort of evidence does the truth of the axiom which affirms that things which are equal to the same thing are equal to one another rest?" we need not hesitate to reply, "On the law of contradiction." Professor Bain thinks not. He claims for it, alone among the axioms, the character of being a generalisation from experience.† "Equality," he says, "is properly defined as immediate coincidence." If it is, why, then, might not the term "coincidence" be used convertibly with the term "equality"? It is plain enough that it could not. If we predicate equality of two lines we do not mean that they do coincide, but that they possess that attribute whereby they would, if superimposed, coincide. The coincidence of two lines would be a matter of fact to which no necessary propositions could ever apply. Their equality is a matter of abstraction, to which such propositions are alone applicable. Coincidence is given by sense, and sense only, and is open to the intelligence of all beings possessed of sight and touch. Equality is learned through sense, of course, but by thought, and is probably quite beyond the intellectual grasp of the Bushman or the Dammara. We cannot use the term "equal" intelligibly without knowing that the equals of equals are equal, any more than we can use the term "black" intelligibly without knowing that what is black is black. Suppose we try to realise the meaning of the negative of the proposition that the equals of equals are equal. We suppose ourselves measuring off any definite length from one line, and then the same length from another line. We then try to put it to ourselves, "Perhaps these lengths, after

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* Ex. Ham. Phil., 4th ed., p. 394.

† "Mental and Moral Science," ed. 3, p. 187.

all, are not the same." If we do, we plainly deny our own assumption that they are the same. The key to the possibility of geometrical demonstration lies in this: in the power that we possess of contemplating one attribute, such as length, as remaining "the same," though in a varied environment. The want of a true theory of identity is indicated, I think, rightly by Mr. Bosanquet ("Mind," li., 3) as being at the root of most of the mistakes of the English school; and this is a case in point. From an identical proposition in the sense of one which affirmed that the length of any given line in any given position is the same now as it was five minutes ago no *geometrical* deduction, at any rate, could be drawn; but with an identical proposition, in the sense of one which affirms that the length of a line can be the same though its position is altered, the case is wholly different. We need no other concession than this to deduce all the properties of the circle. If we inquire, with Spinoza,* what is the efficient cause of a circle, we might answer, with him, "It is the space described by a line of which one point is fixed and the other movable." This line is the radius; and when we conceive of it as the same line, but in varied positions, even Professor Huxley would hardly deny that it would be an identical proposition, with a proof resting on the law of contradiction, to affirm that this line in all its positions is equal to itself. This identity in varied positions would be a not inapt definition of equality. It is a character from which innumerable new truths—of sequence, at any rate—can be drawn; yet it is plain that we arrive at it by the ordinary process of abstraction. When we abstract our attention from the irrelevant circumstances of its position and direction, and contemplate its length alone, we can then, of course, contemplate the length as remaining identical whatever the position and direction may be.

In the Fourth Proposition, which is proved by the method of superposition, Euclid very plainly postulates for the mathematical figures with which he deals this characteristic of being capable of being lifted and moved about and put on top of one another—in other words, of being capable of being regarded as identical in spite of difference of position. This postulate is one which, instead of being taken for granted, ought, I think, to be specifically stated at the beginning of every treatise on Euclid, and clearly kept in view in all geometrical demonstrations. If it were, it would be plain that the construction and the proof resting on it in the Fifth Proposition—the celebrated *pons asinorum*—are mere surplusage. We have in any case to postulate the possibility of taking up the large triangles

* Letter to Schirnhausen.

formed by extension of the sides, reversing one of them, and placing it on top of the other.* Why not at once take up the isosceles triangle itself, reverse it, and superimpose it on its former self? We shall then plainly have two triangles to compare, possessing all the characteristics of those in the Fourth Proposition.

Mr. Mill seems always to speak of the lines and circles of geometry as if they were specimens which we had picked up in our rambles. He would have avoided much confusion of thought if he had contemplated them as what they really are —the lines and circles which we suppose ourselves to have just drawn or to have just constructed.

In the light of the above, let us glance again at what Mr. Mill has to say in regard to the axiom that two straight lines cannot enclose a space. The upholders of the necessity of this proposition, he says accurately enough, uphold it on the ground that we can see its truth by merely thinking of the lines. The answer to this, he thinks, is to be found in the capacity of geometrical forms for being painted in the imagination with a distinctness equal to reality. "Thus," he says, "although we cannot follow two diverging lines by the eye to infinity, yet we know that if they begin to converge it must be at a finite distance; thither we can follow them in our imagination, and satisfy ourselves that if they approach they will not be straight, but curved." That is an accurate description of the process by which we satisfy ourselves that two straight lines cannot enclose a space; but we cannot help asking, Is it a description of the process by which truths of experience are learned? It is one of Mr. Mill's most characteristic doctrines, and one on which he repeatedly and emphatically insists, that† "Whenever we form a new judgment—judge a truth new to us—the judgment is not a recognition of a relation between concepts, but of a succession, a coexistence, or a similitude between facts." Whether we admit that this applies to all new truths or not, we may certainly admit that it applies to all new truths of experience. It is quite plain that a new truth of experience cannot be learned by the process of comparing one of our concepts with another. Yet it is precisely by doing this—by comparing our concept of straight lines with our concept of lines that enclose a space, and finding them incongruous—that Mr. Mill describes us as arriving at the truth that they never enclose a space. His description, in fact, is not a description of the mental process by which truths of experience are learned, but of that

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* That is, of course, taking the proof of Props. IV. and V. together, as Mr. Mill does.

† Ex. Ham. Phil., 4th ed., p. 426.

by which we satisfy ourselves of the truth of identical propositions. We think out the truth of such a proposition; and there could be no better definition of a necessary truth than one which "we see to be true by merely thinking of it." It is obvious, indeed, that we could not have started on our course of mental experiment without being already in possession of all that experience could possibly furnish us with in regard to what straight lines would or would not do in any given circumstances. The fact that we could think out any fresh knowledge about them without reference to the world of fact is itself surely evidence sufficient that such knowledge was already implicated in the more obvious knowledge which we had before us about them, and that all that we required to do in regard to it was to unfold it. Kant's great division, therefore, of *a priori* truths into "analytic" or "implicative," and synthetic or augmentative, seems to be misleading. Analytic or implicative truths may be themselves augmentative. Indeed, if the truths are there *a priori*, though not on the surface, what else can they be but implicated?

To recapitulate, then, we may lay down the following in regard to necessary truths: (1) That they are always concerned with abstractions, never with concrete realities; (2) that the opposite of them is in the strictest sense of the word inconceivable, not merely unbelievable; (3) that this is so because if we think of their opposite we find that the last half of the statement sublates the first; (4) that they are truths which can be seen to be truths by merely thinking about them; and (5) that they are in reality truths of sequence only, not of fact.

The last affirmation brings us face to face with an apparently formidable difficulty. How, it may be asked, can it be that if the truths of geometry are truths of sequence only they can be applied to practical use in the world of fact? Mr. Mill's theory, of course, does not enable him to escape this difficulty. Indeed, he expressly indorses Stewart's view that the truths of mathematics rest on hypotheses. Any difficulty that there is, however, is not peculiar to mathematical reasoning, though it comes out in a more obvious light in connection with it than in connection with other sorts of reasoning. All deductive reasoning, it appears, rests on hypotheses, from the simple affirmation that blue is not green to the latest application of the theory of natural selection in the field of politics or sociology. A fact of sequence, such as the equality of the square on the hypotenuse to the two squares on the other sides of such right-angled triangles as we suppose ourselves to construct, has a real interest and importance for us only when we find that it is, at any rate, approximately true of the right-angled triangles of Nature. Without that, it would have at the best the interest of a game of chess, which is itself a process of necessary reasoning resting on hypotheses, but on hypotheses which have nothing in Nature that corresponds to them.

ART. LXVI.—*The Stability of Ships: its Principles made clear by Models and Diagrams.*

By E. WITHY.

[*Read before the Auckland Institute, 10th August, 1891.*]

Plates L., LI.

RATHER more than twenty years ago public attention was very forcibly called to the question of the stability of ships by the capsizing of H.M.S. "Captain," and the consequent loss of nearly five hundred lives. Not only was the shock produced by this event very great, but the surprise that was generally expressed nearly equalled the shock. The vessel had made two successful preliminary cruises, and was proceeding on a third in company with other men-of-war. She had crossed the Bay of Biscay, and was standing up well to her canvas, when the breeze freshening caused her to list rather more than before, and, without any warning, she steadily settled down, turned completely over, and went to the bottom. Less than twenty men, I believe, escaped and reported the occurrence substantially as I have given it.

Another stimulus was given within the last ten years by the capsizing of the steamer "Daphne" on the occasion of her launch, and the consequent drowning of a number of workmen.

On each occasion an exhaustive inquiry was instituted as to the form and construction of the ships. Naval architects were employed to make such calculations as the existing knowledge of the subject of stability rendered possible. I think I am correct in saying that no addition was thus made to the best information on the subject; but a result of very great importance did follow, and that was, that an enormous impetus was given to the study of the question, and a conviction became wide-spread that, after all, true theory must be more allied with practice than had been the custom.

Both vessels were built by eminent private firms of the first rank, the former on the Mersey and the latter on the Clyde; yet it was evident that these firms did not consider it