Art. XIII.—*On the Causes tending to alter the Eccentricity of Planetary Orbits.*—By Professor A. W. Bickerton.

[*Read before the Philosophical Institute of Canterbury, 6th May,* 1880.]

Plate I.

In former papers it has been shown that the partial impact of cosmical bodies may not unfrequently produce a central mass and attendant bodies, which I have called respectively a sun or nebula, and planets. The sun is at a high temperature and rotates. The planets, in a solid, liquid, or gaseous state, revolve round in one general plane with orbits of varying area and of high eccentricity. All the motions, whether of sun or planets, have one common direction. Further it was shown that the planetary path is due to a portion of the original proper motion escaping conversion into heat at impact. For the same reason the temperature of the planet is lower than that of the sun, whose high molecular velocity, due to its temperature and comparatively small mass, may cause it to expand into a nebula.

The present paper requires that the central mass shall become a nebula, and shall expand beyond aphelion distance of the most remote planet. The forces acting on the planet will be the attraction of the nebula, gaseous adhesion while traversing the nebula, and at the same time exchange of molecules with those of the nebula. The heavier molecules will generally be attracted to the planet, while the lighter ones will leave it. The probability of such a system being formed, or the possibility of gaseous planets moving in a nebula, with its attendant effects on the size of the orbit and the change of apsides, is not treated in this paper. It is solely occupied with the change of eccentricity.

The following are five causes which are calculated to result in such a change:—

1st. An alteration in the amount of the attractive force exerted on the planet by the nebula.

2nd. The varying resistance and interchange of molecules incurred by the planet in its path.

3rd. The gaseous adhesion to the planet revolving on its axis within a nebula.

4th. The accretion of some of the vast number of small bodies which would exist in the nebula.

5th. Some others which are too dependent upon the special character of the impact to be discussed at present.

In compliance with the wishes of several members, I have inserted in this paper the solutions of the dynamical problems involved, whose truth I had before assumed.

The agency of lessened attraction as affecting any one planet, applies only to the period which elapses while the central mass is expanding to a nebula, and it will appear that the first revolution will especially be productive of altered eccentricity on this count. The following shows the action of these forces reduced to geometrical problems:—

Problem 1. Suppose a planet to be at that part of its orbit most distant from the sun, and, while in this position, suppose the mass of the sun suddenly diminished to a given extent,—required to trace the effect of this diminution of the sun's mass upon the orbit of the planet.

At present let the sun's mass be considered constant. Let the line *ax* (fig. 1) be tangent to the curve at aphelion, and *aa, ab, bc* infinitesimals along *ax* in the direction of the planet's course; let *aa'*, *bb'*, *cc'*, be infinitesimals representing the fall of the planet during the times contained respectively in *aa, ab, ac,* then *aa' b' c'* will be the path of the planet.

Now suppose the mass of the sun to be decreased, the infinitesimals *aa, ab, bc* will remain unaltered, but *aa'*, *bb'*, *cc'*, etc., will each be diminished to *a'' b'' c''*. Then the curve *aa'' b'' c''* represents the new orbit. It falls without the old orbit, except at *a* where it coincides with it. Perihelion distance is therefore increased, as represented in fig. 2, by virtue of diminished attraction.

The amount of the lessening of the attractive force will depend upon the quantity of the sun's matter which expands beyond aphelion distance. The portion which so expands ceases to affect the path of the planet. As this increases the orbit will assume variously the forms of the ellipse, circle, ellipse (the foci being reversed), parabola and hyperbola. If the attraction towards the centre entirely ceased, the path would coincide with the line *aa*. These orbits are respectively shown in fig. 2.

In fig. 3 let *p'* represent the orbit with perihelion distance increased beyond that of *p*, this latter representing the orbit if the sun were not to expand into a nebula. Let the dotted circle *c* represent the limits to which the nebula has expanded when the planet passes aphelion. As the planet is entirely in the nebula it will be subject to constantly and rapidly diminishing attraction as it approaches the centre, *s*, hence it will not pass along *p'*, but will move more slowly inwards (in agreement with the first problem), and will pass along the second dotted line *p''*, which shows great increase in perihelion distance.

The two actions which have now been discussed scarcely affect aphelion distance, but render the orbit more circular by increasing perihelion distance.

I have now to notice gaseous resistance and interchange of molecules, whose action will be found chiefly to diminish aphelion distance. The following problem demonstrates decrease of aphelion distance by a resistance at perihelion.

Problem 2. Suppose a planet to be at that part of its orbit nearest to the sun, and, when in that position, suppose a retarding force to act upon it,—required to trace the effect of this upon the orbit of the planet.

Let Px represent a tangent to perihelion, and pa, ab, bc be components in direction pn, passed over in three successive infinitesimals of time. Let $a\ \alpha$, $b\ \beta$, $c\ \gamma$ represent the total fall towards the sun in the same intervals. Then $p\ \alpha\ \beta\ \gamma$, represents the orbit. Now let the velocity in the direction px be diminished by the retarding force, and let the spaces pa', $a'b'$, $b'c'$ represent the components in the direction px in the same infinitesimals of time. The components towards the sun remaining the same draw $\alpha\alpha'$ $\beta\beta'$ $\gamma\gamma'$ parallel to px, then α' β' γ' are points in the new orbit.

This curve lies entirely within the other. Thus, by a retardation at perihelion, aphelion distance is diminished, as shown in fig. 5. If this retardation is great enough, the orbit may become a circle or an ellipse with foci reversed, as shown in fig. 5. The general action of gaseous resistance is to convert the energy of the system into heat by gradually drawing the planet into the sun, or to the centre of attraction. It is maximum at perihelion, for there the density of the nebula is greater than at any other part of the orbit. Molecular exchange results from the varying densities of the different parts of the system. The planets are cooler than the central parts of the nebula, and will most likely be denser than the matter surrounding them in their path, and have sufficient attractive power to collect the heavy molecules in their vicinity. The temperature of the surface of the planet will be raised to an unknown extent by its immersion in the nebula and its progress towards perihelion. Its light molecules have their velocity so increased as to escape the planet, while the heavier molecules of the vicinity, with their lower velocity (though equal temperature), will be attracted, picked up, and become permanently part of the planet. A greater proportion of heavy molecules will be found towards perihelion, for at the centre of the nebula will probably be its greatest density, and the original expansion of the central mass into a nebula will result in the more rapid outward escape of the light molecules compared with the heavy, in obedience to the laws of gaseous diffusion. Thus the accretion of molecules to the planet will be maximum at perihelion distance. Its effect will be to retard the motion of the planet, as, in order to give its own velocity to a molecule, it will impart some of its energy. The escape of the light molecules will not affect the planet's orbit. We find therefore that gaseous resistance and molecular exchange act as resistances to planetary motion and are both maximum at perihelion, thereby decreasing aphelion distance and rendering the orbit more circular.

Gaseous Adhesion.

When a body is moving forward in a gas, the gas adheres and produces retardation. If the body be revolving this retardation is unequal and the body is deflected. The well-known fact that if a projectile revolves on any other axis than its own direction it is deflected, is an illustration of this action, and it is to make the ball move in its true path that a gun is rifled. It has been shown in the paper "On the general problem of stellar collision"* that all the bodies developed by "partial impact" tend to revolve in the same direction; in order, therefore, to ascertain the effect of this gaseous adhesion on the path of a planet revolving in a nebula we have especially to consider the case of a body revolving in the same direction as its orbit, and on an axis perpendicular to its plane.

Problem 3. To ascertain the influence of gaseous adhesion on a rotating planet revolving in a nebula.—Let the arrows in fig. 6 represent the general direction of motion. Let *a b* represent the planet rotating in the direction of its arrow; it is evident that a particle at *a* is tending to move forward faster than a particle at *b*, for if the path of *b* were an epycycle, as it might be, it is evident that for an instant it would be at rest; hence gaseous resistance is stronger at *a* than at *b*, hence *a* is retarded more than *b*, and the direction the body will tend to take is towards *c*. In other words, gaseous adhesion acting on a planet revolving on an axis perpendicular to its ecliptic in the same direction as its orbit tends to straighten the curve.

From the above problem it is evident that on the first return when it meets the nebula it tends to increase perihelion distance and alter apsides, as shown by the dotted ellipse fig. 6. After the first return, were the nebula uniform, it would tend to make a larger ellipse, that is increase its average distance from the centre, thus the potential energy of the planet would be increased, and this increase is done at the expense of the planet's rotation. It might be supposed that this would be a very small matter, but it must be remembered that all the time the body is contracting from a more or less dense gaseous to a liquid state the whole of this potential energy will be converted into rotation, thus the total effect may be very considerable, but as this action will be chiefly at perihelion it will tend materially to alter the eccentricity. It must be clearly understood that it is the differential resistance on the sides of the planet towards and away from the sun that is discussed in this paragraph; its general retarding action was studied in the last problem.

* "Trans. N.Z. Inst.," Vol. XII., Art. XIV.

Fig. 1.

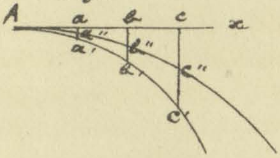


Fig. 2.

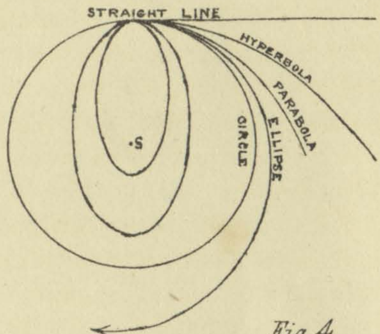


Fig. 3.

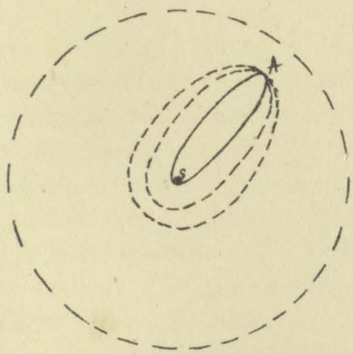


Fig. 4.

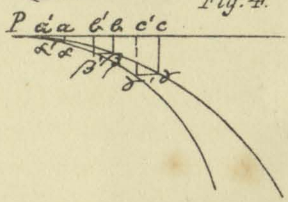


Fig. 6.

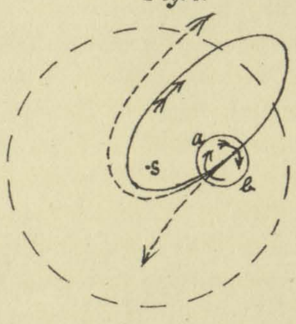


Fig. 5.

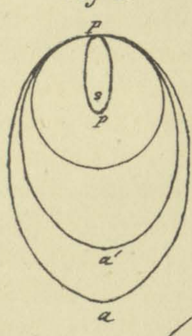
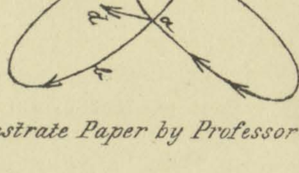


Fig. 7.



To illustrate Paper by Professor Bickerton.

On the Accretion of Particles.

When the nebula has become stable, that is no longer expanding, much of its elementary matter will be at a sufficiently low temperature to combine; and these compound molecules will tend to aggregate into groups, and it may be shown that these masses may ultimately become so considerable as to form star clusters, associated by gravitation but not coalescing. I propose discussing this difficult question in a paper on star clusters; but in this paper I shall simply state that these little bodies will in all probability revolve in independent orbits at all eccentricities around the centre of the mass; these and other masses will be occasionally picked up by the planet.

The following problem shows that component to and from the centre tends to be destroyed, and only a circular orbit left.

Problem 4.—Given two bodies revolving in eccentric orbits in the same direction around an attracting centre, to find the effect upon the eccentricity of the new orbit in the event of their coalescing.—Let *a* represent the two bodies colliding, the direction of one body is along the path *ab*, the other along *ac*, the component along the diagonal *ad* is evidently the new direction of the coalesced body; it has evidently less velocity than the mean of the two, as the component towards and away from the sun is more or less destroyed. The position of the body is also nearer after than before, so that the total effect will be to reduce mean distance, to lessen aphelion distance, and generally to increase perihelion distance, in other words to lessen eccentricity.

There can be but little doubt that this agency of accretion will be most important in giving regularity to any system. Proctor has discussed the influence of accretion of meteors, and it is certain he is right in giving it very great value. It probably played a great part in the formation of Jupiter.

Uncertain Agencies.

An agency whose effect it is difficult to estimate is that of the outward motion of the general mass of the nebula. The planet may meet this on its return towards the centre; if so it will directly oppose its return, acting exactly opposite to gravitation, that is, in the same manner as though the central mass were of less mass; the body will consequently not be attracted back towards the centre so far as it otherwise would be. This action will tend to lessen perihelion distance. This outrush will evidently be much less as the planet leaves the sun on its second revolution, thus the body will not be aided by it on its return, and consequently it may not reach its full aphelion; but it is extremely likely that the nebula will attain equilibrium before it could affect the planet on its return. In the event of a case of partial impact in which the two parts of the original bodies escaped into space—any mass escaping during, or immediately after, the impact would be exposed to a much higher attraction on escaping than on the return, because, on leaving them, the three bodies would be exercizing attraction upon the body; but, on the return journey, there would be only one body. Thus its aphelion may not be near so distant as that of a body that had only the central mass acting on it in its outward journey; but this will not necessarily affect the eccentricity, but may do so in certain cases.

Art. XIV.—*The Origin of the Solar System.*

By Professor A. W. Bickerton.

[*Read before the Philosophical Institute of Canterbury* 5th *August*, 1880.]

The order displayed in the structure of the Solar System strongly suggests the idea that it must have originated in some single event. Laplace has calculated that the probability of such a system having originated in a common cause, is not less than four millions to one. It is evident, therefore, that its origin is a legitimate subject for scientific speculation, and in order to account for the peculiarities of the motions of the planets and satellites, Laplace himself suggested the now well known hypothesis of the release of nebular rings and their subsequent coalescence. This theory has found many supporters, but when it is examined in the light of the doctrine of the conservation of energy and the dynamical theory of gases, so many difficulties present themselves as to throw great doubt upon it, in fact, as Denison says, it has been so little accepted by English mathematicians that it has scarcely been discussed, and Faye has recently given his opinion that it must be given up. A modification of the theory is offered in Newcombe's "Astronomy,"—it is that the release of rings commenced on the inside.

I hope to examine a number of the difficulties of these theories in a future paper. Proctor has discussed the probability of the system having been formed by the coalescence of an immense number of meteorites, and to this hypothesis there appear to be fewer objections than to the other two. In fact it is highly probable that such an action has aided materially in giving symmetry to the system. But as the sole agent in the formation of the solar system these suggestions have two great objections:—the extreme slowness of the sun's present rotation; and the irregularities in the system, such as the eccentricity of the orbits, the inclinations of the axes and orbital planes, and retrograde motions. It cannot therefore be considered that a satisfactory solution of the problem has been given, and it is probable from its extreme