the English alphabet is treated in the way in practice: when the child is taught that *a* = *ae*, and no symbol is given for the broad *a*; that *i* = *ai*; that *u* = *iu*, &c. Let the reform begin at the fountain head, by a re-arrangement of the alphabet.

One or two Scotch names give good examples of the difficulties in spelling brought about by the want of system in English orthography. Let us take the name MacNeil. We find this variously spelt McNeil and McNeal. Although apparently a Celtic name, I suspect that it came from Scandinavia, where we have to this day the frequent Christian name of *Nil, Nils*. The French could make nothing of Neil, so changed the spelling to Niel, in the case of the celebrated marshal. The McNeils and Neals should do the same, and the name would then be written phonetically.

We find the name Mackay spelt the same, whether the owner of it comes from the Highlands or from Galloway; but the pronunciation is different. In the former case it is *Mackai*, in the latter *Măckae*; and at San Francisco I found another variation, viz., *Mackāe*, the accent being on the last syllable.

In looking up the Scandinavian languages, I have been struck with the similarity in some respects to broad Scotch, and I suspect that the language of the old kingdom of Northumbria, extending from the Humber to the Forth, has been more influenced by Scandinavian immigrants than is generally supposed. Such words as *baru* for *bairu* are suggestive; and in Norwegian I found a sentence, viz.: "*Qua sae?*" meaning "What do you say?" which one may hear any day in the streets of Edinburgh or Glasgow.

Art. VII.—*The Non-Euclidian Geometry Vindicated: a Reply to Mr. Skey.*

By F. W. Frankland F.I.A.

[*Read before the Wellington Philosophical Society, 13th February*, 1884.]

The following observations are an abridgment of a series of letters addressed to Mr. Skey, the author of the paper entitled "Notes upon Mr. Frankland's Paper 'On the Simplest Continuous Manifoldness of two Dimensions and of Finite Extent,'" read before the Wellington Philosophical Society on 26th June, 1880, and contained on pages 100–109 of the thirteenth volume of the Transactions of the New Zealand Institute. By Mr. Skey's kindness and courtesy these letters were made available to me for the preparation of a printed reply to his criticisms. I make no apology for the form in which this reply appears. I have taken, *seriatim*, the main points which Mr. Skey raised, and replied to each of his contentions in detail. Mr. Skey's own words are in each case placed at the commencement of the paragraph, and the number of the page from which the quotation is made is indicated. It seemed to me that in this way only could a searching and exhaustive refutation of his arguments be given.

1. What is meant by the assertion that "the axioms of geometry may be only approximately true"? (p. 100) It means that the actual physical constitution of the space in which we live may be different from the space treated of in works on solid geometry, but that it must be *so nearly* the same that we cannot detect the difference by the most delicate experimental methods at our command.

2. "The author then adverts to 'the existence' of a particular manifoldness, which has been treated by Professor Clifford in a lecture on the postulates of space" (p. 101). I mean it *exists* in the sense of being logically constructible, not in the sense that any surface in the space in which we live possesses such properties. It *may* be that planes (or *flattest surfaces*, if the expression be preferred,) in the space in which we live possess the properties of this "manifoldness." We cannot know whether they do or not. If they do, at any rate their total areas must be immensely large.

Perhaps it may be said that any absurd scheme of pseudo-geometry is "logically constructible." But this is not the case. It is not possible, for instance, to construct a scheme of geometry in which two shortest lines enclose a space (all shortest lines being supposed congruent), and in which the three angles of a triangle are always less than two right angles. Such a scheme would be logically self-contradictory. For it is logically involved in the assertion that two shortest lines may meet twice, assuming all Euclid's other axioms to be true, that the three angles of a triangle are always *greater* than two right angles. They *cannot*, under such circumstances be either equal to 180° or less than 180°.

3. "Then he describes how this space is analytically conceived, with the object of putting us in a position to apprehend certain discoveries of his own, which relate to its very singular properties" (p. 101). The manifold* I described in my paper is not a *space*. It is a manifold of *two* dimensions, not of *three*. It may be described as an unimaginable but logically constructible *surface*.

4. It is not accurate to say that Professor Clifford "imputes finiteness" to the universe or to space. He says, in common with most living mathematicians who have studied this question, that space *may* be finite—not that it *is* finite. Its possible finiteness is spoken of, not in the sense of its having a *boundary*, which would be unmeaning, but as implying that space may return into itself, so to speak, just as the surface of a sphere and

* This term is now generally used instead of the more cumbrous "manifoldness."

the circumference of a circle return into themselves. In other words, the totality of space may have a finite volume, just as the surface of a sphere has a finite area, and the circumference of a circle a finite length. As far as pure mathematics go, we cannot decide whether space is infinite or finite. *Experience* alone can decide; or, rather, although we cannot imagine any experience sufficiently extensive to prove the infinitude of space, experience may possibly some day prove its finiteness.

5. "The prime object" of the paper "is to spread and support the views of the metaphysical school." . . . . "This view is supported by the fact, that just recently this gentleman has read before us a very able and profound paper, entitled, 'Mind Stuff,' and which is evidently of a highly metaphysical character" (p. 101). The allegation here quoted is so far from being correct, that I claim for my paper on "Mind Stuff" the character of complete consistency with the experiential philosophy. It endeavours to show that the only things of which we have any direct knowledge are the feelings we ourselves experience. By a legitimate inference from experience we conclude that there is a world outside us which causes these feelings, and this world I infer to be composed of stuff ("mind stuff," Professor Clifford called it,) remotely similar to our own feelings, but not worked up into so complex a structure. If by the "metaphysical school" be meant the school which holds that we can discover truth otherwise than by experiment and observation, then it is precisely the school which the non-Euclidian geometry has done more than anything else to confute. The geometry of Euclid has hitherto been their stronghold: "Here, at least," they have hitherto said, "the human mind can, without any appeal to experiment, evolve, from its own structure, truths which hold good with absolute exactness, throughout immensity and eternity." Now, since the researches of Lobatchewsky and Gauss this can no longer be said. They and their successors have conclusively shown that, as far as logical consistency is concerned, there are an infinite number of alternative geometries, and that experience alone can decide which of these is physically true.

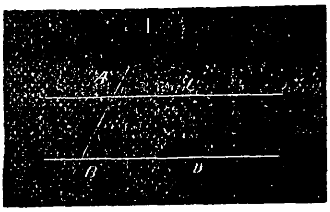
6. To the expression "geometers of the Euclidian school" (p. 101) I take exception, believing that none such are left in the sense in which Mr. Skey uses the word. The triumph of the non-Euclidian geometry, or, I will say, the "general" geometry, has been complete. I can safely appeal, on this point, to any distinguished member of any Mathematical Society in Europe or America.

7. "It is not this equivalent which Lobatchewsky is supposed to use in his attempt at demonstrating the truth of his assumption" (p. 102). Neither Lobatchewsky nor any one else has attempted to demonstrate the *truth* of the assumption, but

only to demonstrate *that no one else can demonstrate its falsity.* In other words, he has attempted to demonstrate (and that he has completely succeeded all modern mathematicians allow) that the truth of Euclid's 12th axiom can by no possible succession of syllogisms be deduced from the other axioms and the definitions of the straight line, plane, parallels, &c. Innumerable attempts had been made to do this—*i.e.*, to put the 12th axiom on the same logical footing as, for instance, the 5th proposition of the First Book. All the attempts had failed. Lobatchewsky proved, once for all, that they must necessarily fail, by constructing an unimaginable but perfectly self-consistent scheme of geometry, in which all the other axioms were assumed to be true, and all the definitions remain the same, but in which this one axiom (the 12th) was assumed to be false. The equivalents of Euclid's axiom which I have mentioned are really exact logical equivalents. If one is true, all are true. If one is false, all are false. In Euclid's space all are true: in Lobatchewsky's, all are false.

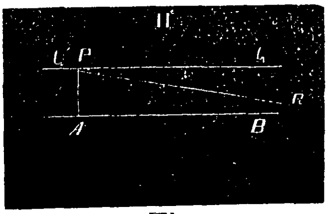
8. I propose now to establish the exact logical equivalence of the three forms of the parallel-axiom mentioned in my paper.

Form (a), (Euclid's) is:—"If a straight line meets two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced shall at length meet upon that side on which are the angles which are less than two right angles." In other words, if the angle $C\ A\ B$ + the angle $A\ B\ D < 180°$, then $A\ C$ and $B\ D$ will at length meet.



This is Euclid's axiom, and it is to my mind just as good as any of its modern substitutes.

I now propose to deduce from this axiom the usual modern substitute:—"It is impossible to draw more than one straight line parallel to a given straight line (*i.e.*, lying in the same plane with it, but not intersecting it) through a given point outside it." Let $Q\ P\ A + P\ A\ B = 180°$. Then, by a proposition of Euclid which does *not*, directly or indirectly, rest on the 12th axiom, $P\ Q$ can never intersect $A\ B$.



Draw any straight line $P\ R$ within $Q\ P\ A$. Then,

Since $Q\ P\ A + P\ A\ B$ = two right angles

$\therefore$ $R\ P\ A + P\ A\ B <$ two right angles.

$\therefore$ $P\ R$ will eventually meet $A\ B$ (Euclid's 12th axiom), *i.e.*, $P\ R$ cannot be parallel to $A\ B$. Hence no line within $Q\ P\ A$ and passing through P can be parallel to $A\ B$.

Similiarly, no line through P and passing outside $Q\ P\ A$ can be parallel to $A\ B$, for the continuation of it would fall within the angle $Q'\ P\ A$. Hence only one straight line can be drawn through P parallel to $A\ B$, viz: $P\ Q$. · *Q.E.D.*

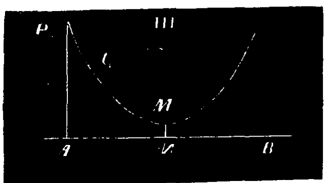
I have thus shown that if Euclid's axiom is true, then the modern substitute is true. To establish the exact logical equivalence of the two axioms, I should have to prove the converse formally, viz.: that if the modern substitute is true, then Euclid's axiom is true. But I assume it will be conceded that the above reasoning can quite well be put in the converse form. I now pass to the third equivalent, which is alleged by Mr. Skey not to be a real equivalent of the other two. If it be borne in mind that the word *parallel* in the second equivalent means *not* equidistance along the whole length of two lines; but *lying in the same plane, plus non-intersection however far produced* (see Euclid's definition)—if it be borne in mind that I define parallelism in this way, I think it will be recognised at once that the second and third forms of the axiom are merely two different ways of saying the same thing.

However, as truth and falsehood in nature can never be dependent on the signification of *words*, I may as well say how the axiom would be worded if we define two straight lines to be parallel when they are equidistant along their whole length. (I vastly prefer this definition, though it is not the usual one.) Taking this as the definition of parallelism, Euclid's axiom may be stated thus:—"Two straight lines lying in the same plane, and not being parallel, (*i.e.*, not equidistant along their whole length,) must ultimately intersect if sufficiently produced in both directions."

In Lobatchewsky's geometry, on the other hand, straight lines in a plane need not intersect though they are *not* equidistant along their whole length. They may approach each other for awhile, reach a minimum mutual distance, and then recede more and more continually. Also in Lobatchewsky's geometry *no* two *straight* lines can be parallel, in the sense of being equidistant along their whole length. If two lines are parallel (*i.e.*, equidistant along their whole length), they cannot both be straight. One, at least, must be a *curved* line, *i.e.*, a longer line than some other which could be drawn through any two of its points.

9. "Nothing is said as to the distance away from this line at which the point is to be placed" (page 103). (This quotation refers to the point outside the first line through which the second line is drawn.) The distance of the point from the line may be as short as possible, and still (if Euclid's 12th axiom is untrue) there will be a finite angle through which the rotating line can be turned without *ever* intersecting the fixed line: the magnitude of this angle depending partly on the distance of the

point and partly on the nature of the space under consideration (*i.e.*, on the degree to which the space deviates from the properties of the ideal space of Euclid). For there are spaces *and* spaces which satisfy Lobatchewsky's conditions. There is only one space which satisfies Euclid's conditions, but there is an infinite number satisfying Lobatchewsky's. They vary through infinite gradations, from one which has such feeble "negative curvature" that it can hardly be distinguished from Euclidian space, to one which has such strong "negative curvature" that even $P\,Q$ (in the annexed figure) would not meet $A\,B$, but would rapidly come to its point of minimum distance ($M\,N$), and would then recede for ever from $A\,B$.



Now, in regard to the space we actually live in, we ought, in my opinion, to say this: "It may be Euclidian, or it may have negative curvature: but if it has negative curvature, that curvature must be excessively weak, though not *infinitely* weak, as is suggested." Professor Clifford puts the case very well in his lecture on "The Aims and Instruments of Scientific Thought." He says: "Suppose that three points are taken in space, distant from one another as far as the sun is from *a* Centauri, and that the shortest distances between these points are drawn so as to form a triangle, and suppose the angles of this triangle to be very accurately measured and added together: this can at present be done so accurately that the error shall certainly be less than one minute, less therefore than the five-thousandth part of a right angle. Then I do not know that this sum would differ at all from two right angles; *but also I do not know that the difference would be less than ten degrees, or the ninth part of a right angle. And I have reasons for not knowing.*"

Clifford introduces this example by saying, what requires to be much insisted on, that these speculations on non-Euclidian space are not merely questions of words, as many people imagine, but that the issue involved is "a very distinct and simple question of fact." In plain language, geometry is a *physical* and *experimental* science, just as much as optics or physiology; and the properties of space cannot be evolved from man's inner consciousness, but must be determined by *experiment* and *observation.* There was as much justification, before the curvature of the earth was known, for erecting into an axiom the proposition that all verticals are parallel—(For myself, I cannot, even now, *imagine* its falsehood, although I of course *know* it to be false)—as there is now for the statement, *a priori*, that two shortest lines cannot enclose a space, or that the three angles of a triangle are exactly equal to two right angles.

10. " . . . it appears to me that even if the angle of convergence is infinitely small the lines would intersect, but not, of

course, at any determinable or conceivable distance" (p. 103). This is beside the question. The true question is, whether they will necessarily intersect if the angle is, for instance, one decillionth of a degree. Those who regard the Euclidian geometry as absolutely true, must hold that they will. Modern mathematicians, on the other hand, say that we do not know whether they will or not. Who can prove that they will? Euclid frankly admitted that he could not, by *assuming* the alleged fact as his twelfth axiom. Since Euclid's time, scores of mathematicians have tried to prove it, but all their attempted proofs are justly regarded by their fellow-mathematicians as simply inconclusive. It *cannot* be proved. Experiment cannot prove it; reasoning has failed to prove it: our intuitions—if, as disciples of the experiential school of philosophy, we believe they have been produced by the experience of our ancestors through millions of years in the portion of space passed through by our solar system in that time—cannot be trusted as infallible, and, therefore, cannot prove it. Lastly, it will not be contended that any supernatural revelation has been vouchsafed on this point.

11. "None of the evidence of Lobatchewsky in favour of this is given by Mr. Frankland" (p. 104). It did not fall within my province to give this evidence. It is to be found in Lobatchewsky's works. The evidence is admitted, and has long been admitted, to be conclusive by all mathematicians who have studied the question. Also, I think I may fairly add that the burden of proof lies with those who say that an intersection must and will take place, not with those who say that it may or may not take place.

12. "It appears to me that at any finite angle of convergence of $C D$ to $A B$ they will intersect at some determinable part of the line $A B$, *for a finite angle can only mean an angle of such a size* that it can be measured or conceived of." Just so: it can be measured by the ratio of a finite arc (subtended by the angle) to the radius of the same circle. But this does not prove that it must be measured by a portion of the straight line $A B$. How, then, does it follow as a "necessary corollary" that "there is a point along $A B$ which the line P will pass through?" (p. 104.) It will hardly be considered a proof to say that "It seems that the completion of the ideal construction thus begun demands this intersection" (p. 103). If this can be proved, the most remarkable advance in geometry since the time of Euclid himself will have been made. A whole literature has grown up in the attempt to furnish this proof. Its growth has been arrested by the discoveries of Lobatchewsky and Gauss, and I feel very sure that the desired proof will never be forthcoming.

13. Mr. Frankland (p. 106, *note*) "gravely informs us here, that the finishing point or goal for a geodesic line in process

of construction is to be the length of such a line away from the starting point of that line. The two points are to be apart, yet coincide!" Where is the contradiction? In the manifold I describe, as on the surface of a sphere, a geodesic starting from any point leads back eventually to that point. So far, my manifold and the surface of a sphere resemble one another. The difference is this: If two persons on the surface of a sphere (say the earth) were to start from the same place, and travel along geodesic lines, they would cross each other's paths at a half-way house (on the other side of the sphere), and then again at the starting point. But on the manifold I have investigated they would, after travelling a certain distance, get back to the starting point, *but without ever having crossed each other's paths in the meanwhile.* On a Euclidian plane, on the other hand, they would obviously never either cross each other's paths or get back to the starting point at all.

14. "Geodesic lines, then, proceeding from some common point of a surface, are to diverge somehow from the polar of that point" (p. 106). I do not know what Mr. Skey means by the "polar of that point," unless, indeed, it be the *opposite* point. If so, I reply that in my manifold, which for the future we may for convenience call the "finite plane,"* a point has not *one* opposite only (like a point on a sphere), but a whole row of opposite points: that is to say, an opposite *line.* The geodesic lines proceeding from a common point cut this "opposite line" (which I have called the polar) in *separate points, each of which* is equally "opposite" to the common centre of radiation.

15. "He is assuming a uniformly curved surface of immense size" (p. 106). By no means. The manifold may be of any size, large or small. Its total area may be less than the decillionth part of a square inch—yet it will have its complete and thoroughly self-consistent, though, I admit, quite unimaginable, geometry. What I do say is that, if any surface *constructible in the space in which we live* possesses the properties of a "finite plane," then that surface must be of immense size, for we can prove by experiment that no closed surface of *moderate* area constructible in our space does possess these properties.

16. "It is manifest that the analytical conception of two geodesic lines refusing to intersect each other more than once, and so enclosing but one space, is founded upon Lobatchewsky's conception of what parallel straight lines are capable of" (p. 106). This is not so. It is founded on just the opposite conception. Lobatchewsky's conception is that of two geodesic lines which, even though converging at first, do not ultimately intersect; mine is that of two geodesic lines which ultimately intersect,

* The manifold in question possesses the same properties as the "plane at infinity," well known to students of solid geometry.

even though divergent at first. In Lobatchewsky's space the three angles of a triangle are always together less than two right angles: in the "finite plane" (and also in the corresponding space of three dimensions,) they are always greater than two right angles, just as the angles of a spherical triangle are. In Lobatchewsky's space, figures have their edges and corners sharpened when their linear dimensions are proportionately increased: in the "finite plane" they have their angles blunted on being magnified, (like the figures on a sphere,) and in the corresponding space of three dimensions solid figures would also have their edges and corners blunted on being magnified.

17. "It is, I think, abundantly evident that the analytical conception of a surface such as the one which has been worked upon for the discoveries communicated in his (Mr. Frankland's) paper, is not, in reality, valid; and that though possibly not self-contradictory, as he urges, it requires premises which *are* of this nature"—*i.e.* self-contradictory (p. 107). Not so. The premises are not self-contradictory, but only contradictory to some of our strongest and firmest intuitions—viz., our space-intuitions. But so is the convergence of verticals, already alluded to, and yet it is an unquestionable fact. Believing, as I do, that our space-intuitions are derived simply from ancestral experience, aided by natural selection (which must always have tended to eliminate those in whom such intuitions were relatively weak), I can only admit that they are reliable enough *for practical purposes;* not that they are exactly true through all space and time. The parallelism of verticals was an intuition, (a sort of dynamical intuition,) ingrained in our mental constitution by ancestral experience through innumerable generations. Were we blind, and confined (say by surrounding climates of excessive rigour) to a very limited area of the earth's surface, I think it very likely that this conception would to this day seem to us self-evidently true. It would seem as certain that two verticals must have the same direction as it now does that two shortest lines cannot enclose a space. A Skey, in such a world, might even have argued that to construct a system of cosmography in which two verticals should not have the same direction would be, "though possibly not self-contradictory," to assume "premises which are of that nature." In any case, I do not think that any self-contradiction can be shown to be involved in the proposition that two geodesic lines, though finite in length, intersect only once.

18. "Referring to the idea that the universe is of finite extent," . . . the Professor "argues that 'in this case the universe is again a valid conception . . . for the extent of space is a finite number of cubic miles'" (p. 107). In this quotation from Professor Clifford, two important words

are omitted. The original reads thus:—"In this case the universe, *as known*, is again a valid conception," &c. Professor Clifford very clearly explains what he means by this, in an earlier part of the lecture from which I quoted. Referring to the state of science before Lobatchewsky he says, " . . . the laws of space and motion that we are presently going to examine, implied an infinite space and an infinite duration, about whose properties as space and time everything was accurately known. The very constitution of those parts of it which are at an infinite distance from us, 'geometry upon the plane at infinity,' is just as well known, if the Euclidian assumptions are true, as the geometry of any portion of this room. In this infinite and thoroughly well-known space the universe is situated during at least some portion of an infinite and thoroughly well-known time. So that here we have real knowledge of something at least that concerns the cosmos; something that is true throughout the immensities and eternities. That something Lobatchewsky and his successors have taken away. The geometer of to-day knows nothing about the nature of actually existing space at an infinite distance: he knows nothing about the properties of this present space in a past or a future eternity. He knows, indeed, that the laws assumed by Euclid are true with an accuracy that no direct experiment can approach, not only in this place where we are, but at places at a distance from us which no astronomer has conceived: but he knows this as of here, and now; beyond his range is a there, and a then, of which he knows nothing at present, but may ultimately come to know more. So, you see, there is a real parallel between the work of Copernicus and his successors on the one hand, and the work of Lobatchewsky and his successors on the other. In both of these the knowledge of immensity and eternity is replaced by knowledge of here and now. And in virtue of these two revolutions the idea of the universe, the macrocosm, the all, as a subject of human knowledge, and therefore of human interest, has fallen to pieces."

Well, then: If space should turn out to be of finite extent, the idea of the universe (the universe of *matter* at any rate) would be reinstated, as in a certain measure an object of knowledge throughout its entire extent, as it was supposed to be before Lobatchewsky arose, when Euclidian geometers could tell us the exact constitution of the whole of space.

19. "To make the conclusion agree with the premises, it should have gone no further than to affirm that the universe may not differ *sensibly* from an infinite one" (p. 108). By no means: The surface of a sheet of still water does not differ sensibly from a Euclidian plane, but the surface of the Pacific Ocean, even if perfectly calm, differs very sensibly from a plane,

The imperceptible divergence of small portions from the ideal standard is *cumulative,* and when we take very large portions the divergence accumulates to a very perceptible amount. The difference between the geometry of a cubic mile, if Euclid's assumptions are true, and the geometry of a cubic mile if they are false, we know, by experiment, to be quite insensible: yet by the accumulation of excessively small (though not infinitely small) divergences, it comes about that the geometry of a decillion cubic miles (*i.e.*, 10^{60} cubic miles) may be so different on the two hypotheses, that while, if Euclid's assumptions are true the decillion cubic miles are but an infinitesimal portion of entire space, if his assumptions are false, all space may actually not hold so large a number of cubic miles.

20. "The Professor, having perchance, after all, some doubts as to the validity of this deduction, or possibly forgetting he has *proved* it, essays to prove it again; he says, 'and this (finiteness of the universe) comes about in a very curious way'" (p. 108). I can assure my critic that Professor Clifford had no such doubts. If the universe is such that two shortest lines may enclose a space, and if, nevertheless, all the other assumptions of Euclid are true, then the extent of space is *certainly* a finite number of cubic miles. The one statement is logically involved in the other, though it may require a long and intricate process of reasoning to prove it so.

21. "The qualification put upon straight lines, '*straight according to Leibnitz,*' put, no doubt, all in good faith, as explanative of straight lines, it does still, I feel assured, confer upon them properties which straight lines have not" (p. 108). It undoubtedly confers upon them properties which Euclidian straight lines have not; but the lines in question, though not Euclidian straight lines—and if you will, not *straight* lines at all, for the quarrel need not be over a word when the issue is one of fact—may nevertheless be the straightest lines that can possibly be constructed (even ideally) in the space in which we actually live. In other words, space may be so constituted that what Euclid calls straight lines cannot possibly be constructed in it, any more than a straight line can be constructed on the surface of a sphere. Nevertheless the straightest lines constructible may be of the same shape all along *and on all sides,* which great circles of a sphere are not: for though of the same shape all along, they are concave on the one side and convex on the other, also they may be shortest lines, which the great circles of a sphere are not, relatively to solid space. The quarrel about the definition of a straight line does not affect the issue in the smallest degree.

22. "I blame making so much, in this way, of the gap 'in the chain of reasoning,' by which the truths of geometry should be logically connected and represented" (p. 109). They *cannot*

all be logically connected. Not one, but several, unproved assumptions must be made before a definite geometry can be constructed. The difficulty does not arise from shortcomings in the definitions, though these are undoubtedly defective. Frame what definitions we please, we must still assume certain *matters of fact*, or *alleged* matters of fact (call them axioms or call them postulates), before we can logically raise the superstructure of the Euclidian geometry. Even if we define straight lines and planes as such lines and surfaces that the propositions of Euclid respecting straight lines and planes shall be true respecting them, even by this extreme procedure we get no nearer the desired goal: for it then remains to be proved that straight lines, planes, parallels, &c., *exist in the space in which we live.* To assume that they do is to assume a whole congeries of axioms. A writer named Thomson once wrote a book called "Geometry without Axioms," which was certainly a desperate effort to get rid of unproved assumptions. The attempted proof of the redoubtable 12th axiom was a perfect labyrinth of intricate propositions; but, like all similar efforts, like any efforts which may be hereafter made to ground geometry on definitions and dispense with axioms, it was but "as the helpless waves that break upon the iron rocks of doom."

The science of the space in which we live is a *physical* and *experimental* science, and, unlike arithmetic, algebra, and all the branches of mathematical analysis (the general theory of manifolds among them), cannot be evolved out of man's inner consciousness.

Art. VIII.—*On a new Form of Seismograph.*

By F. Bull.

[*Read before the Wellington Philosophical Society, 23rd September*, 1885.]

The prevalence of earthquakes in New Zealand, and at the same time the uncertainty in the reports from the different parts of the colony, as to their occurrence and direction, owing to the want of proper instruments for their detection, led me to consider the possibility of devising an apparatus which would at once place on record the occurrence of shakes and indicate their direction. Accordingly I set to work, and commenced by planning all sorts of complicated machines, which did not at all satisfy me; and I eventually came to the conclusion that the most simple and direct-acting machine would be the best for the purpose.

The first plan I then adopted was to suspend a heavy sphere of lead, having on its under-side a small tube, fixed vertically, in which a pencil fitted, with freedom to ascend and descend in