striking any plane surface at a fair right angle, but exhibiting infinitely more irregularity in its course than even a body of water; how easily that may be deflected it is needless to state, the fact is obvious to any one who has watched the current of a tide or river. This attribute of water is, *à fortiori*, that of the more elastic fluid—air—and hence it is that I think it open to grave doubt whether, even if we grant that the wind may move at the *immense* velocity asserted, it really exerts that dynamical pressure, on a given plane area, which, *cæteris paribus*, we should be led from such velocity to predicate.

This paper already has exceeded due limit as to length, and I must defer the further investigation of the subject to a second paper.

Art. XXVIII.—*Elements of Mathematics.* By James Adams, B.A.

[*Read before the Auckland Institute, 4th September, 1876.*]

When Peter the Great determined to rouse his subjects to the active life and business habits of the people of England and Prussia, he began by removing impediments. He wished his people to become skilful workmen and mechanics; and it was evident that the Russian of his time, with his long flowing robe and his pendulous beard, could not work at the forge or the bench. To remedy this, Peter stationed men at the city gates, each armed with a pair of shears, who cut off the long skirts and sacred beard of all those who passed through the gates.

This was the first step in giving them a mechanical education, and the effect he produced, in raising his people to the level of other European nations, has always been a subject of admiration.

A similar course was adopted, in the matter of education, after the French Revolution; when the School Commissioners dismissed, in a summary manner, the teachers of the schools and colleges, and flung after them, so to speak, the golden legends, controversial treatises, Aristotle's Ethics, and Euclid's Elements; not that they felt no reverence for these books, but because a *new* era had arrived, when practical knowledge had taken the place of speculative, and when it was of paramount inportance that the students should reach, by the shortest and plainest route, the wide range of learning that was now for the first time opened to the human mind.

The object of education to their mind was to study the nature of things, with the view of adding to the comfort and happiness of man, and not to learn to dispute in the argumentative manner of ancient philosophers.

They confessed their inability to comprehend how that acumen of intellect could lead to the *summum bonum*—

> "Which could distinguish and divide
> A hair 'twixt south and south-west side."

It was fully believed by the supporters of the old system, that such a change would banish learning from the earth, and that, like Orpheus, it would be torn to pieces by the Bacchanalians. But as no such consummation took place—but, on the contrary, that literature, and science, and art flourished with renewed vigour—the denunciations ceased, and similar changes in the school system were adopted in other countries. England, since that time, has been slowly making changes. Greek and Latin verses are not now composed with the same assiduity as formerly. The rules of the Latin Grammar are written in English; the *as in presenti* is not so commonly learned, and many other changes have been grudgingly made with the view of cutting off the non-essential, and thus affording more time for the pupils to study physical science and mathematics.

The English have not followed the example of the French in discarding Euclid's Elements, but, on the contrary, they have made it a standard text-book for those who intend to study the higher mathematics. And an admirable book it is, as it contains the summary of what was known of mathematics during that brilliant period of history, when Ptolemy Soter ruled Egypt; and not only so, but geometry cannot be known without the proof of Euclid's propositions. But, that it should be absolutely necessary to use Euclid's own words in the proofs, now that the range of mathematics is so much wider and the aim so different, is open to grave doubt.

The difficulty a pupil experiences in entering on the study of geometry is great enough, by his having for the first time to form conceptions of quantities of two dimensions without adding any unnecessary obstacle. The actual work he ought to accomplish is quite enough without leading him to it by a most circuitous route. This work may be thus arranged:—

First, to calculate areas from line measurements.

Second, to calculate areas and distances by means of measured lines and angles.

The third step takes in the additional element of force, but the object is still to determine some point or points in space. This embraces the usual mathematical course for secondary schools, namely—mensuration, trigonometry, and mixed mathematics.

It is evident that, in order to learn the first, mathematically, the pupil must solve a great many geometrical problems, understand the principles on which constructions are obtained, and acquire the method of calculation. But as all this leads to a definite object, he learns only what he shall absolutely require, and of which he must constantly make use.

Now, it is thought necessary for a boy to know, at least, two books of Euclid before he can properly commence to survey with the chain alone, and as Euclid, like an ancient philosopher, is speculative rather than practical in his propositions, the abstract relations of magnitudes are alone regarded, and none of the particular or general terms used in calculation are at all mentioned. The result is that, after a pupil has mastered the two books, he cannot, of himself, discover anything in them relating to calculation of areas. And besides, his attention has been directed to the demonstration alone, and the essential part of construction is left to his own invention. Euclid's long proofs and admirable chains of reasoning are not put to practice after the books are read, for, in actual questions, the proofs are written in the algebraic form.

Again, it is found that some of Euclid's propositions are made unnecessarily difficult, some of them are self-evident, and none of them expressed in modern mathematical language.

Take, as an example, the fifth in the first book, called the *pons asinorum*, which is a real barrier to many learners, and was at one time the limit to geometrical studies for the generality at the universities. Now this propoposition can be proved like the fourth by super-position, so that the dullest can at once comprehend it; but, as teachers must conform to Euclid and Euclid alone, two weeks of school life, allowing two hours a-week for geometry, must be spent before the *most intelligent* boys can comprehend it.

The name that this fifth proposition has obtained shows the vast number who have failed to pursue the study of mathematics owing to their inability to see the proof veiled, as it is, in Euclid's drapery of words.

The eighth proposition is another stumbling-block, owing to the indirect method, although the theorem can be far more easily proved directly, and at the same time render the seventh proposition unnecessary. The thirteenth proposition really needs no proof, and for this very reason boys find great difficulty in writing down or saying Euclid's proof for it; and such instances can be multiplied.

It is nothing new to be aware of the faults in Euclid's Elements, but the range of mathematics and physical science was so limited until the eighteenth century, that the time might be spared for Euclid's proofs, as there was little more to learn.

We know what took place when Euclid undertook to teach Ptolemy Soter. After that the latter had learned the definitions, many of which are more difficult than the things defined—had passed safely through the postulates and axioms, and arrived, as we may suppose, at the *pons asinorum*—the King asked him if there was no easier method. Euclid gave the reply so often repeated, "There is no royal road to geometry." I think I may venture to say that there are excellent navigators, surveyors, and engineers, who could not prove this proposition in Euclid's manner; and if there was no shorter method we should not have attained to our present knowledge of mathematics.

For the second step in geometry, namely, to commence the study of trigonometry, it is usual to learn six books of Euclid. This means four years at least of careful teaching in a secondary school; for, as our primary schools are, as a rule, of a most inferior description, nearly all the elementary work must be done in the secondary schools. Four years are spent before the pupil enters on the study of mathematics in such a form as to induce him to pursue the study after leaving school. Now, after these books are mastered, the pupil finds that he has not learned the language of trigonometry, nor the method of the proofs.

It is as if he had learned Latin in order to speak French. He will have acquired such terms as *invertendo, convertendo, ex æquali,* and *ex æquo,* duplicate and triplicate ratios; but not a word of *sines* and *cosines,* nor even the relative values, in general terms, of the sides of a triangle to each other, nor of the side of a regular figure inscribed in a circle to the diameter. The abstract proofs of Euclid confuse rather than clear his understanding, when he has to calculate areas and distances by general or by concrete values. The natural order of instruction is the concrete first, then the general, and last of all the abstract; but in teaching mathematics we reverse the order. Let anyone who knows Euclid's Elements, as now read commence Plane Trigonometry, and I feel sure he will be astounded at the little preparation he has made. He will find that the work of the modern mathematician is with definite values, and with every variety of new problems, of which the construction must be found, and moreover that the proof is in the algebraic form, quite regardless of Euclid's language.

When it is considered that every branch of mathematics has its own language or terminology—and this is the real difficulty in trigonometry, conic sections, mechanics, hydrostatics, and every other subject—what is more reasonable than to dispense with Euclid's language, which does not apply to our modern methods of calculation? If we had a text book, with the single definite object of preparing pupils to enter on the study of trigonometry, there would be a great deal of valuable time saved. Nor is there any fear of speculative geometry dying out, as those with only a taste for mathematics are too prone to it, but in the greater mathematicians it amounts almost to a disease. Mathematical proofs cannot be otherwise than rigid, whether we use the modern or the ancient method. But to solve a problem by the analytic method, and then write out the proof in the synthetic is exactly what Macaulay charges Samuel Johnson with doing, namely, first writing in English and then translating into Johnsonese.

The object of mathematics is calculation, and they become extended just in proportion as the method of calculation is improved.

Now it is worth while to consider the facilities that existed for calculation in Euclid's time, and for many hundred years afterwards. Arithmetic, as we now know it, was then in its infancy. In fact, the seventh, eighth, and ninth books of Euclid's Elements are devoted to this important subject; but Euclid has written with such obscurity that his most devoted worshippers do not insist upon our reading these books. The clumsy symbols that were employed effectually hindered progress, and no one but a philosopher could multiply fractions. It would repay the trouble to work out a few sums in the Greek method, which continued to be employed until the so-called Arabic symbols and method were introduced from the east. Fractions, that enter so largely into our arithmetical calculations, have not been long properly understood. Killand and Tait, in their preface to Quaternious, give a curious instance of the conceptions entertained of them in the sixteenth century. At the present time we can solve all questions by them, and thus dispense with the so-called rules of arithmetic. In fact arithmetic cannot be taught as a branch of mathematics, unless by the aid of fractions, which enable us to keep the whole question before the mind at the same time. But Euclid had no such method of expressing the ideas in his learned head, and so he expressed them in the best manner he could. Where he speaks of multiples we use fractions, and his equality of ratios are with us equality of fractions. Duplicate, compound, and triplicate ratios lose their learned and formidable appearance when we employ fractions.

It is scarcely credible that Euclid would have devoted so many words to prove propositions if he had our concise method of recording results. If, for instance, we know that the area of a triangle is half the product of the base by the perpendicular, we must see that, when the perpendicular is constant, the areas of triangles vary as the length of the bases. Yet we know what scaffolding Euclid has erected in order to prove this self-evident proposition, and his proof is most difficult for learners to fully comprehend. As an example of the contrast between the modern mode of proof and Euclid's, the nineteenth proposition of the sixth book may be taken. It is required to prove that "similar triangles are to each other in the duplicate ratio of their homologous sides." Most of us, in thinking over the proof in the Elements, will remember the number of anxious students who could not understand the proof, but a little progress in algebra makes them reason thus:—Since the triangles $a\ b\ c$ and $a'\ b'\ c'$ are similar, the perpendiculars (p and p') on the sides b and b' from the vertical angles will divide the triangles into others which are respectively similar. Then $\frac{a}{b} = \frac{a'}{b'}$ and also $\frac{a}{p} = \frac{a'}{p'}$. Multiply these equal fractions and $\frac{a^2}{bp} = \frac{a'^2}{b'p'}$ or $\frac{a^2}{a'^2} = \frac{\frac{1}{2}bp}{\frac{1}{2}b'p'}$.

Thus the areas of similar triangles vary as the squares of the sides opposite the equal angles.

The books of Euclid are read by pupils, as commanded, but as soon as they read the chapter on ratio in the algebra they adopt the algebraic method, because the mind always takes the shortest route to a conclusion, and this appears to be the reason that self-evident propositions present so much difficulty to beginners.

The great fault in Euclid is that the pupil is not allowed to know as much about the proposition as the instructor.

The process by which Euclid arrived at his proof had been known to philosophers at least from the time of Plato. It is called the analytic method in geometrical researches, for by this method the problem is supposed to be solved, and then by comparing the magnitudes under this new aspect, and observing the relation between those given and those sought, a way to solve the problem is discovered. This is algebra as applied to mathematics, and it is the grand method of invention. All the progress made in mathematics during the eighteenth century is owing to it.

Pascal and Roberval made use of it; but when they had solved the problem they wrote out the demonstration in Euclid's synthetic method, and designedly concealed their method of invention.

Sir Isaac Newton did the same thing, not that he desired to hide the method by which he arrived at the solution, for even with this aid mathematics are not very easy, but he considered a proof was unfit for publication unless given after the manner of Euclid, and clothed as far as possible in his language. The following is a quotation from his work on *Fluxions* :—

"*Postquam area alicujus curvæ ita* (analytically) *reperta est et constructa, induganda est demonstratio constructionis ut omisso quatenus fieri potest calculo algebraico theorema fiat concinnum et elegans ac lumen publicum sustinere valeat.*

It appears from this that in such a question as the following taken from Todhunter's Mechanics—"Find the centre of equal like parallel forces acting at seven of the angular points of a cube"—that it is not sufficient to determine the point, which is easily done by supposing the forces to act at all the angular points; but Newton considered such a method unfit to see the light, and that the position of the point must be shown by geometrical construction; and this is often so difficult that the exercise may well commend itself to the great mathematicians as a kind of mental gymnastics. But for a teacher who is anxious to conduct a pupil in the study of mathematics to appoint from whence he can see their value and their beauty, it is most injudicious to perplex the learner's mind with intricate questions which properly belong to the speculative philosopher, and very often also of such a nature that the solution, synthetically, is easily discovered by a more extended knowledge of mathematics.

In spite of the opinion of such a genius as Newton, the algebraic method is adopted of necessity in all mathematical books of a practical nature; and scientific men adopt the same method. But discarded as Euclid's method and language are, for all practical purposes, there still lingers a conviction of their all-sufficiency for school-boys just as Mavor's spelling-book is the *sine quâ non* of primary instruction. Our own University, that should take into account the drawbacks that boys here experience in acquiring details, as they are deprived of the advantages of morning and evening study, which boys at home possess, insists on the very words of Euclid. On the top of the paper on Geometry is printed, "No symbols used to denote algebraic operations are to be employed in this paper." Which appears to mean, "Do not employ the signs used in practical mathematics, but the blessed, blessed words of the great Euclid." There is a great waste of time, as I have already shown, in learning to clothe a geometrical proof in Euclid's cumbrous drapery of words, which is a hindrance in entering on the actual study of mathematics. In fact, boys cannot compete creditably in the mathematical examination fixed by the same University for senior scholarships—if they must first go through so much to learn so little. If it is really intended that boys shall learn mathematics, we ought to have a text book of the *elements of modern* mathematics, and not the elements of the mathematics known 2,200 years ago; or at least the public examiners should require no other propositions than those bearing directly on trigonometry, and the proofs to be expressed in the usual mathematical symbols.

Anyone acquainted with trigonometry will perceive that this change would no more do away with the study of geometry than that the Russians all died from having their coat-tails cut off; but, on the contrary, there would be five mathematicians to one at present.

Euclid's elements properly belong to the department of logic, and to that department the study of them should be confined in our public examinations.

ART. XXIX.—*On some points connected with the Construction of the Bridge over the Grey River at the Brunner Gorge.* By C. H. H. Cook, M.A.; Fell. St. John's Coll., Cam.; Prof. Math., Cant. Coll.

[*Read before the Philosophical Institute of Canterbury, 5th October, 1876.*]

It is not my intention to examine into the cause of the disaster which overtook the suspension bridge at the Brunner Gorge on the morning of the 28th