cause is influential to the extent of about 4 per cent. Hence the bird will not preserve a horizontal flight, if the velocity falls below 100 feet per second, without increasing its angle of flight more than the assumed 7°. Another slight error occurs in Captain Hutton's calculations which is probably an oversight. Using his equation $H\,E = \dfrac{30}{\tan\ A\,E\,H + \sin\ C\,E\,H\,.\,\cos C\,E\,H}$ he makes $H\,E = 115$, when $A\,E\,H = 0$, and $C\,E\,H = 15°$. The true result is 120.

When proper data have been obtained, the solution of the problems connected with this "sailing flight" should, I think, be approached in an entirely different manner from that adopted by Captain Hutton. His deductions as to the resistance of the air to a projectile of the form of the albatros are of no value at all, and may, I think, be shown to be inconsistent with facts already ascertained. The principal portion of the resistance is that which is resolved into a sustaining, or upward bearing, force, and this is exerted against the obliquely exposed under surfaces of the bird. The formula for the resistance of a fluid to a plane, moving obliquely through it is—

$$R = \tfrac{1}{2}\,Q\,v^2\,\sin^3 I.\,A$$

where Q is the density of the fluid, v the velocity of the stream *plus* that of the plane if it is moving against it, I the ∠ of inclination to the stream, and A the area of the plane. The two latter coefficients will have to be determined from observations, which in the case of I it will be very difficult to make. If the part of R which is resolved into a force sustaining the bird against gravity be known, let this = C, then the *retarding* force of atmospheric resistance against the inclined surfaces of the body and wings,—

$$= \tfrac{1}{2}\,Q\,v^2\,\sin^3 I.\,A - C$$

We must deduct this quantity from the whole retardation observed to find what would be the resistance to the front surfaces of the bird when both body and wings were horizontal. It is only by this process that we can obtain a quantity which is comparable with the atmospheric resistance to round shot.

Art. LX.—*On Sinking Funds.* By Captain F. W. Hutton, F.G.S.

[Read before the Auckland Institute, September 7, 1868.]

The subject of Sinking Funds is one of much importance to this and other countries, but I have not been able to find any book that treats of it, and I therefore think that an investigation of its principles may prove both useful and interesting.

By "Sinking Fund" is meant a sum of money put away annually in order to pay off a loan. There are two principal ways in which this money is applied: either it may be invested year by year until, with the interest accruing on it, it amounts to a sum sufficient to pay off the original loan; or else it may be used to take up yearly a portion of the loan until the whole has vanished. I propose to investigate both these methods, and then compare them together.

The second case, where the fund is applied yearly to buy up the loan, is very simple.

Let a equal the amount of the loan.
" p the amount put by as Sinking Fund each year, and
" T equal the number of years it will take to pay off the loan. Then

it is evident that as $\frac{1}{p}$ of the loan is brought up every year,

$$T=\frac{a}{p} \quad . \quad . \quad . \quad (1)$$

$$p=\frac{a}{T} \quad . \quad . \quad . \quad (2)$$

$$a=p\,T \quad . \quad . \quad . \quad (3)$$

With regard to the first case: let a and p be as before, but let t be the number of years it will take to pay off the loan by this method, and let v equal 1 + the interest on one pound at the rate at which the Sinking Fund is invested, so that if it is invested at 5 per cent. it will equal 1.05.

Now at the end of the

1st year the fund will amount to $p\,v$

2nd " " " $(p+p\,v)\,v = p\,v+p\,v^2$

3rd " " " $(p\,v+p\,v^2+p)\,v = p\,v+p\,v^2+p\,v^3$

tth " " " $p\,v+p\,v^2+p\,v^3+\&c. \ldots\ldots p\,v^t$

but at the end of the last or tth year, the fund must equal *a*

$$\therefore \quad p\,v+p\,v^2+p\,v^3+\&c. \ldots\ldots p\,v^t = a$$

multiply by v and

$$p\,v^2+p\,v^3+p\,v^4+\&c. \ldots\ldots p\,v^{t+1} = a\,v$$

subtracting

$$p\,v^{t+1}-p\,v \quad = \quad a\,v-a$$

$$\therefore \quad a\,(v-1)=p\,v\,(v^t-1). \quad . \quad . \quad . \quad (4)$$

$$\therefore \quad a=\frac{p\,v\,(v^t-1)}{v-1} \quad . \quad . \quad . \quad . \quad (5)$$

$$p=\frac{a\,(v-1)}{v\,(v^t-1)} \quad . \quad . \quad . \quad . \quad (6)$$

When p is known the per centage required for forming a Sinking Fund equal to p can be found by multiplying p by 100 and dividing by *a*.

$$\text{From (4) we get } v^t-1=\frac{a\,(v-1)}{p\,v}$$

$$\therefore \quad v^t=\frac{a\,(v-1)+p\,v}{p\,v}$$

$$\therefore \quad t\log v=\log\left\{a\,(v-1)+p\,v\right\}-\log p\,v$$

$$\therefore \quad t=\frac{\log\left\{a\,(v-1)+p\,v\right\}\log p\,v}{\log v} \quad . \quad . \quad (7)$$

$$\text{From (4) we also get } p\,v^{t+1}-(a+p)\,v+a=0 \quad . \quad (8)$$

From which v can be found by the following rule, known as Bernoulli's.

1. Find by trial two numbers nearly equal to t.
2. Substitute these assumed numbers for t and mark the error that arises from each with + if too great, and − if too small.
3. Multiply the difference of the assumed numbers by the least error, and divide the product by the difference of the errors when they have like signs, but by their sum when they have unlike.
4. Add the quotient to the assumed number belonging to the least error when that number is too little, or subtract if too great.

5. This operation may be repeated until t is found sufficiently near.

I will now take the total amount of interest that has to be paid on the loan until it is all taken up.

This on the first system will evidently be a t (v′−1), where v′ is 1+ the interest that has to be paid on one pound of the loan for a year.

On the second system the interest payable at the end of the

1st	year would be		(a−p) (v′−1)
2nd	,,	,,	(a−2p) (v′−1)
3rd	,,	,,	(a−3p) (v′−1)
(T−1)	,,	,,	(a−(T−1) p) (v′−1).

But the year before the whole loan was taken up only $\frac{1}{p}$th of it would be left, it is evident that a−(T−1) p=p

So that we have an equidifferent series of which (a−p) (v′−1) is the first term, p (v′−1) the last, and T 1 the number of terms. The sum of them therefore, or the whole interest to be paid on the loan

$$\frac{T-1}{2}\left\{(a-p)\ (v'-1)+p\ (v'-1)\right\}$$

$$=\frac{a\ (v'-1)}{2}(T-1)$$

Therefore

$$a\ (v'-1)\ t : \frac{a\ (v'-1)}{2}(T-1) :: \left\{\begin{matrix}\text{amount of interest}\\ \text{by first method}\end{matrix}\right\} : \left\{\begin{matrix}\text{amount of interest}\\ \text{by second method}\end{matrix}\right.$$

Or 2 t : T−1 :: . . . . : . . . . .

But besides the interest on the loan there has also to be paid for the Sinking Fund by the first method p t pounds, and by the second method p T pounds. So that

$$p\,t : p\,T :: \left\{\begin{matrix}\text{amount paid for Sinking}\\ \text{Fund by first method}\end{matrix}\right\} : \left\{\begin{matrix}\text{amount paid for Sinking}\\ \text{Fund by second method}\end{matrix}\right.$$

And combining the two we get

$$2\,t+p\,t : T-1+p\,T :: \left\{\begin{matrix}\text{whole amount paid}\\ \text{by first method}\end{matrix}\right\} : \left\{\begin{matrix}\text{whole amount paid}\\ \text{by second method}\end{matrix}\right.$$

Or (p+2) t : (p+1) T−1 :: . . . . : . . . . . .

Now the limits of p are o and a, and as it gets small both T and t increase, but t will increase slower that T for it also depends upon the value of v which remains stationary. On the contrary as p gets large t will decrease more slowly than T for the same reason, and the position of equality will of course depend upon the values of v and a. If however we take $a > 1000$; $p < \frac{a}{14}$, and $v = 1.05$—which in practice will include all cases—it will be found that

$$(p+2)\ t < (p+1)\ T-1.$$

The actual amount that would have to be spent by either method can be easily found by substituting in the following formulæ the different values for a, p, v, and v′.

$$\text{By the first method} \left\{ a\,(v'-1)+p \right\} \cdot \frac{\log \left\{ a\,(v-1)+p\,v \right\} - \log p\,v}{\log v.} \cdot$$

$$\text{By the second method} \; \frac{a\,(v'-1)\,(a-p)}{2\,p} + a.$$

From this comparison it follows that when money can be invested at 5 per cent., and the Sinking Fund is less than 7 per cent. of the loan, the first is the more economical method; and the smaller the Sinking Fund, and the higher the rate of interest, the greater will be the saving effected by investing the fund in other securities, than by using it to buy up annually part of the loan.

This however is only the mathematical or pecuniary view of the question; from the political point of view many reasons can be given why the second method should be preferred, and the difference pecuniarily is not sufficiently great to override them.

ART. LXI.—*List of Plants found in the Northern District of the Province of Auckland.* By J. BUCHANAN and T. KIRK.

[IN the course of the geological survey of the above district in 1865-6, an extensive collection of plants was made by Mr. Buchanan, and forwarded to Dr. Hooker, at Kew. They were, however, unfortunately, distributed by an assistant without being examined, so that a complete list was not obtained, and any few novelties escaped notice in the appendix to Vol. ii. of the "Handbook of the New Zealand Flora."

From the portion of the collection retained, and from notes made on the spot, Mr. Buchanan compiled the greater part of the following list, with the exception of the natural orders, Junceæ, Restiaceæ, Cyperaceæ, and Gramineæ, the lists of which are furnished altogether by Mr. Kirk. As Mr. Buchanan collected in the months of November and December, and Mr. Kirk went over most of the same ground in April, the latter observer was also able to add largely to the number of plants, the results, as combined in the following lists, should give a tolerably complete Flora of each locality indicated.

An account of the chief plants of interest obtained by Mr. Kirk is given in a paper published in the "Transactions" for last year (p. 140); along with which his contribution to the following tables was to have been printed, had not circumstances prevented it.

For the characteristic plants of the district, and a comparison of its botany with that of other parts of New Zealand, the reader is referred to Mr. Colenso's Essay, also in Vol. i. of the "Transactions."—ED.]

Introductory Remarks by J. BUCHANAN.

The above area may be divided into eight districts, viz.:—

1. Wangarei,
2. Bay of Islands,
3. Wangaroa,
4. Stephenson's Island,

the latter as showing the comparative botany of a portion of land detached from the Main Island.

5. Mount Camel,
6. North Cape.