

5. Find the relation which must exist between the coefficients a , b , and c , in order that the expression $ax^2 + bx + c$ may be a complete square.

6. Extract the square roots of $l^{2x} + l^{-2x} - 2$ and $\frac{9}{4}a^3 - 5a^{\frac{5}{2}}b^{\frac{1}{2}} + \frac{179}{45}a^2b - \frac{4}{3}a^{\frac{3}{2}}b^{\frac{3}{2}} + \frac{4}{25}ab^2$

7. Solve the equations—

$$(a.) \quad \frac{2x+a}{b} - \frac{x-b}{a} = \frac{3ax+(a-b)^2}{ab}$$

$$(b.) \quad \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}} = \frac{b}{a}$$

$$(c.) \quad \frac{x+1}{y} = \frac{3}{5} = \frac{y-1}{x}$$

$$(d.) \quad \begin{cases} 6x + y + 8z = 50 \\ 8x + 3y - 10z = -6 \\ 12x + 7y - 16z = 2 \end{cases}$$

8. Divide a number a into three parts, such that the first may be to the second as m is to n , and the second may be to the third as p is to q .

9. Three persons divide a certain sum of money in the following manner: A takes one-third of the whole together with £8; B takes one-third of the remainder together with £8; C takes one-third of what now remains together with £8; and then nothing remains. Find the sum.

10. If the numerator and denominator of a certain fraction be each increased by 2, the value of the fraction will be $\frac{3}{5}$; but if each of them be diminished by 3, the value of the fraction will be $\frac{2}{3}$. Find the fraction.

Euclid.—For Class D, and for Junior Civil Service. Time allowed: 3 hours.

1. Define an angle, a circle, a rhombus, parallel straight lines, a parallelogram.

2. If two triangles have the sides of the one respectively equal to the sides of the other, prove that the triangles are equal in all respects.

Show that if the opposite sides of a quadrilateral are equal the opposite angles are also equal.

3. Prove that the sum of any two sides of a triangle is greater than the third side, and that the difference between any two sides is less than the third side.

4. Prove that the opposite sides and angles of a parallelogram are equal, and that the diagonal bisects its area.

Prove that any straight line through the intersection of the diagonals of a parallelogram bisects its area.

5. If the square on one side of a triangle be equal to the sum of the squares on the other two sides, show that the angle contained by those two sides is a right angle.

6. If a straight line be divided into any two parts, the sum of the squares on the whole line and one of the parts is equal to twice the rectangle contained by the whole line and that part, together with the square on the other part.

7. Prove that in any triangle the square on the side subtending an acute angle is less than the sum of the squares on the other sides, and show by how much.

The sides of a triangle are respectively 5 in., 6 in., and 8 in.: what kind of a triangle is it?

8. ABC is an equilateral triangle, and D is any point in the side BC: prove that the square on BC is equal to the rectangle contained by BD, DC, together with the square on AD.

Euclid, Books I.—IV.—For Senior Civil Service. Time allowed: 3 hours.

1. Show that if one side of a triangle be produced, the exterior angle is greater than either of the interior and opposite angles, and any two of the angles of the triangle are together less than two right angles.

Enunciate the subsequent proposition in which these results are virtually included.

2. Give Euclid's definition and postulate for parallel straight lines.

Two straight lines, AB, CD, are met by another at E, F, and a pair of alternate angles are equal: show by superimposing AEF on DFEB that AB is parallel to CD.

Prove also the converse of this theorem.

3. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram is double of the triangle.

If two triangles have the rectangle contained by the base and the altitude of the one equal to the rectangle contained by the base and the altitude of the other, the triangles are equal.

If the straight line joining the vertices of two triangles on the same base be bisected by the base or by the base produced the triangles are equal to each other.

4. Describe a square equal to a given rectilineal figure.

Describe also a right-angled isosceles triangle equal to the same rectilineal figure.

5. If straight lines are drawn to the circumference of a circle from any point which is not the centre, they are in the same order of magnitude as the angles they subtend at the centre.

Hence show that if more than two equal straight lines can be drawn from a point to the circumference, that point is the centre.

6. Show how to draw a tangent to a circle from a given point either on or without the circumference.

Show also how to draw the common tangents of two circles.