5. Prove the relations-
(a.) $\quad \sin (\mathrm{A}+\mathrm{B}) \cdot \sin (\mathrm{A}-\mathrm{B})=\sin ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~B}$;
(b.) $\frac{\sin A+\sin 3 A}{\cos A+\cos 3 A}=\tan 2 A$;
(c.) $\frac{1-\tan ^{2}\left(45^{\circ}-\mathrm{A}\right)}{1+\tan ^{2}\left(45^{\circ}-\mathrm{A}\right)}=\sin 2 \mathrm{~A}$.
6. Find an expression for $\sin (A+B+C)$ in terms of the ratios of $A, B$, and $C$.

If $\mathrm{A}+\mathrm{B}+\mathrm{C}=180^{\circ}$, show that-

$$
\sin ^{2} \mathrm{~A}+\sin ^{2} \mathrm{~B}+\sin ^{2} \mathrm{C}=2(1+\cos \mathrm{A} \cos \mathrm{~B} \cos \mathrm{C}) .
$$

7. If A be the angle of a triangle, find $\cos \mathrm{A}$ and $\cos \frac{1}{2} \mathrm{~A}$ in terms of the sides.

If the sides of a triangle are $12 \mathrm{ft} ., 16 \mathrm{ft}$., and 20 ft ., find the greatest angle and the area of the triangle.
8. Show how to solve a triangle when two sides and the contained angle are given, and give proofs of the formulæ employed.

Prove that in any triangle

$$
a \cos \mathrm{~A}+b \cos \mathrm{~B}=c \cos (\mathrm{~A}-\mathrm{B}) .
$$

9. Given $\log 2=\cdot 301$, and $\log 3=\cdot 477$, find the logarithms of $6,15, \cdot 18$, and $\frac{1}{12}$.

Find also L $\sin 60^{\circ}$.
10. A steamer at sea sighted a lighthouse bearing due west, and after the steamer had proceeded on a west-north-west course for sixteen miles the bearing of the lighthouse was found to be south-west: find the distance of the steamer from the lighthouse at the time of each observation. [Given $\sin 22 \frac{1}{2}^{\circ}=3827$.]

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