

Algebra.—For Senior Civil Service. Time allowed: 3 hours.

1. If $a = 1$, $b = 2$, $c = 3$, find the values of—

(1.) $a^3 + b^3 - c^3 + 3abc$;

(2.) $\frac{\sqrt[3]{a^2 + b^2 + \frac{1}{3}c^2}}{\sqrt[3]{a^2 + 9b^2 + 3c^2}} + \frac{\sqrt[3]{a^3 + 7b^3 + c^3 - 3}}{\sqrt[3]{8a^3 + 9b^3 + 240c^3 + 1}}$.

2. State the rule of signs in multiplication. Prove that the smallest numerical value of $x + \frac{1}{x}$ is 2.

3. Multiply $5x^3 + 4x^2 + 3x + 4$ by $5x^3 - 4x^2 + 3x - 4$; and divide the product by $5x^2 + x + 4$.

4. Define the highest common divisor of two algebraical expressions. Find the highest common divisor of $5x^3 + 4x^2 + 3x + 4$ and $25x^4 + 39x^2 + 16$.

5. Find the lowest common multiple of the expressions given in the last two questions.

6. Define a fraction. State what must be the meaning attached to the multiplication of one fraction by another in order that $\frac{a}{b} \times \frac{c}{d}$ may equal $\frac{ac}{bd}$.

7. Simplify:—

(1.) $\frac{5x^3 + 4x^2 + 3x + 4}{25x^4 + 39x^2 + 16}$.

(2.) $\left\{ \frac{3x-2}{3x+2} + \frac{3x+2}{3x-2} + \frac{8}{4-9x^2} \right\} \div \left\{ \frac{3x+2}{3x-2} - \frac{3x-2}{3x+2} \right\}$.

(3.) $\left(a - \frac{x^2 - ab}{a - b} \right) \left(b + \frac{x^2 - ab}{a - b} \right) + \left(\frac{x^2 - ab}{a - b} \right)^2$.

8. State the axioms which are used in the solution of a simple equation.

Solve the equation—

$$\frac{3x+2}{5} + \frac{7x+4}{11} = \frac{8x+5}{13} + \frac{7x+3}{10},$$

pointing out at each step the axiom employed.

9. Solve the equations:—

(1.) $\frac{ax+b}{cx+d} = \frac{ax+p}{cx+q}$.

(2.) $\frac{16-x}{2} + \frac{(x-8)^2}{2(x-4)} + \frac{12}{4-x} = 0$.

(3.) $1 + \frac{1}{x + \frac{1}{x}} = \frac{17}{25}$.

10. Find the square root of $25x^4 + 10x^3 + 41x^2 + 8x + 16$. Show that your process *proves* that the square of the result is equal to the original expression.

11. A certain quantity of carpet is required to cover the floor of a room: if the width of the room were 2ft. less, and the length 1ft. more, 4 yards less of carpet would be required: if the room were 2ft. wider and 4ft. longer, it would take 20 yards more than it actually does: the width of the carpet being 2ft., find the length and breadth of the room.

12. At a certain post-office the total postage for the mail, consisting of local letters which are charged 1d., general letters charged 2d., and letter-cards at 1½d., is £10: if the numbers of local letters and letter-cards were interchanged the total sum would be £9 15s. 10d.; while if, the number altogether remaining the same, each letter and letter-card had cost 2d. the cost would have been £10 16s. 8d.: find the number of each.

Euclid.—For Class D, and for Junior Civil Service. Time allowed: 3 hours.

1. Show that if two triangles have two sides of the one equal to two sides of the other, each to each, and have also the angles contained by those sides equal, then the triangles are equal in every respect. Show that the line that bisects the vertical angle of an isosceles triangle bisects the base at right angles.

2. Show that if two straight lines intersect one another the opposite angles are equal. What is the converse of this?

3. Show that if two lines be drawn to a point within a triangle from the extremities of its base their sum is less than the sum of the remaining sides, but they contain a greater angle.

4. State Euclid's axiom respecting parallel straight lines, and enunciate the proposition which Euclid establishes by the aid of this axiom. Show that straight lines which are parallel to the same straight line are parallel to one another.

5. Show that if two equal triangles be upon the same base the line joining their vertices is parallel to the base if the triangles are on the same side of it, and is bisected by the base if they are upon opposite sides of it.