Euclid, Books I.–IV.—For Senior Civil Service. Time allowed: 3 hours.

1. Show how to bisect a given rectilineal angle. In the construction why is the triangle described on the side of the base-line remote from the given angle?

If ABC be the given angle, and if CB be produced to D, and the angles ABC, ABD, be bisected, prove that the bisectors are at right angles to one another.

2. Given three lines, any two of which are together greater than the third, construct a triangle with its sides severally equal to the three given lines.

Explain where, in your construction, the necessity for the limitation of the lengths of the given straight lines comes in. Illustrate your answer by figures showing what may happen if the limitation be removed.

3. If the square described on one side of a triangle be equal to the sum of the squares on the other two sides, the angle contained by these sides is a right angle.

4. If a straight line be divided into any two parts the square on the whole line is equal to the sum of the squares on the parts together with twice the rectangle contained by those parts.

Prove this proposition also by showing that it can be deduced at once from the two preceding propositions.

5. If a quadrilateral can be inscribed in a circle the opposite angles are together equal to two right angles.

Any two circles cut one another in A and B: if through A and B two parallel straight lines PAQ, XBY, be drawn, terminated by the circles, these lines are equal to one another.

6. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles made by this line with the line touching the circle shall be equal to the angles in the alternate segments of the circle.

7. Prove that any equiangular pentagon inscribed in a circle is equilateral.

Trigonometry.—For Senior Civil Service. Time allowed: 3 hours.

1. What are the different units employed in measuring angles? Investigate equations connecting the measures of the same angle in terms of these units.

A piece of string is laid along the circumference of a circle a yard in diameter, and subtends at the centre an angle of 25°: find its length.

2. Define the tangent of an angle, and trace the changes in its value as the angle increases from zero to two right angles.

Prove from a figure that $\text{Sec}\,(A-90^\circ)=\text{Cosec}\,A$.

3. Prove the following relations:—

$$(\text{Sin}\,60^\circ-\text{Sin}\,45^\circ)(\text{Cos}\,30^\circ+\text{Cos}\,45^\circ)=\text{Sin}^2\,30^\circ.$$

$$\frac{\text{Sin}\,A}{1+\text{Cos}\,A}+\frac{1+\text{Cos}\,A}{\text{Sin}\,A}=2\,\text{Cosec}\,A.$$

$$\frac{\text{Cot}^2x-\text{Cos}^2x}{\text{Tan}^2x-\text{Sin}^2x}=\text{Cot}^6x.$$

4. Prove that—

$$(\text{Sec}\,3A+\text{Sin}\,A)\,\text{Sin}\,A+(\text{Cos}\,3A-\text{Cos}\,A)\,\text{Cos}\,A=0.$$

$$\frac{\text{Sin}^3A-\text{Cos}^3A}{2+\text{Sin}\,2A}=\frac{1}{\sqrt{2}}\,\text{Sin}\,(A-45^\circ).$$

5. Explain how logarithms may be employed to abridge the labour of raising a number to a power, giving the necessary proof.

Write down the following: $\text{Log}_{\sqrt{2}}8$; $\text{Log}_8 4$; $\text{Log}_{25}125$.

If $\text{Log}_{10}2=a$; $\text{Log}_{10}3=b$; $\text{Log}_{10}7=c$; find the logarithms of 105; 4·2; $\frac{\cdot 006}{7}$.

6. Prove that, in any triangle ABC, $\text{Cos}\,C=\frac{a^2+b^2-c^2}{2ab}$; and, assuming similar formulæ for the cosines of the other two angles, deduce

$$\frac{\text{Sin}\,A}{a}=\frac{\text{Sin}\,B}{b}=\frac{\text{Sin}\,C}{c}.$$

Prove also that $a\,\text{Cos}\,A+b\,\text{Cos}\,B+c\,\text{Cos}\,C=2a\,\text{Sin}\,B\,\text{Sin}\,C$.

7. The elevation of a tower due north of a station at A is α, and at a station B due west of A is β: prove that the altitude is $\frac{c\,\text{Sin}\,\alpha\,\text{Sin}\,\beta}{\sqrt{\text{Sin}^2\alpha-\text{Sin}^2\beta}}$ where c is the distance between A and B.

Mechanics.—For Class D, and for Senior Civil Service. Time allowed: 3 hours.

1. Define *acceleration, work, momentum, moment, couple, poundal, level, specific gravity.*

2. Give the rules for compounding two uniform rectilinear velocities.

Two trains, 250ft. and 300ft. long, are travelling at 15 and 30 miles an hour in opposite directions. How long do they take to pass one another?

3. Enunciate Newton's Laws of Motion.

4. A bullet is fired vertically upwards with an initial velocity of 1,600ft. per second. Find its height and velocity after 20 seconds.