

*Arithmetic.—Alternative with Algebra for Senior Civil Service. Time allowed: Three hours.*

1. A cubic foot of water weighs 1,000oz., and a gallon contains 277·274 cubic inches: how many gallons will weigh a ton, and what is the weight of a pint?

2. A certain kind of silver plate costs 7s. 8d. an ounce in London; but if it is to be exported, a drawback of 1s. 6d. an ounce is allowed. A New Zealand purchaser, after paying freight and duty, the latter at the rate of 15 per cent. on the net price of the silver to him, finds that his silver has cost him 2d. an ounce less than if he had bought it for use in London. Find what percentage of the original cost he has paid in freight.

3. A rectangular field is three times as long as it is wide, and the length of the diagonal is 1,240ft.: find the area of the field.

4. A can run a mile in 4min. 40sec. and B can run a mile in 4min. 50sec. In a mile race B is allowed 50 yards start. Which will win, and by what distance?

5. One publisher supplies booksellers at 25 per cent. below the publishing price, and, in addition, gives 13 copies of each book for the reduced price of a dozen; another sells at 30 per cent. below the publishing price: which publisher is it most advantageous to deal with?

Suppose a retailer buys from the latter of these two publishers, and sells at an advance of 10 per cent. on the publishing price, giving six months' credit, at a time when money is worth 8 per cent.; calculate his actual gain per cent. on his outlay.

6. The population of a place increases by one-twentieth of itself every year: if at present it be 1,940,400, what was it two years ago, and what will it be two years hence?

7. Show that the square root of a whole number must be either a whole number or an interminable decimal which does not recur.

8. The Three per Cent. Consols are quoted at  $103\frac{1}{2}$ , and a person has £10,000 which he sells out, and sends the proceeds to New Zealand, where it is invested on mortgage at 7 per cent: if all capital in the colony pay a penny in the pound per annum for property-tax, and the cost of transmitting money either to or from New Zealand be  $1\frac{1}{2}$  per cent., find by how much the capitalist has increased his annual income by sending his money here for investment.

9. Two trains are travelling on parallel lines of rail; one is going at the rate of 15 miles an hour, and the other at the rate of 25 miles an hour: prove that they take four times as long to pass one another if they travel in the same direction as if they travel in opposite directions.

If one train be 66 yards long and the other 110 yards long, calculate what these times are.

10. A square building which has a frontage of 50ft. has a roof consisting of four equal equilateral faces: find the height of the apex of the roof above the top of the walls, and the number of square feet in the area of the roof.

11. If 1,000lb. can be carried 1,000 miles for £1, and the rate of conveyance be the same in France, find to two places of decimals how many kilogrammes can be carried 100 kilometres for 20 francs, having given £1 = 25·2 francs, 1 kilometre = ·6214 mile, and 1 kilogramme = 2·2046lb.

12. The attractions which the earth and the sun exert on the moon are directly proportional to the masses of the two former bodies and inversely as the square of their distances from the moon. If the distance of the sun from the moon be 400 times that of the earth, and the mass of the sun 320,000 times that of the earth, compare the attractions which they exert on the moon.

*Algebra.—Alternative with Arithmetic for Senior Civil Service. Time allowed: Three hours.*

1. Twice the number  $b$  is subtracted from the number  $a$ , and the excess is divided by three times the number  $c$ ; from this result is subtracted five times the number which when multiplied by itself gives the quotient obtained by dividing the number  $a$  by the number  $b$ ; the difference so obtained is multiplied by itself. Write down the algebraical expression for the above operation.

2. Multiply together  $ax^3 + 3bx^2y - 2cxy^2 - 5y^3$  and  $2py^2 - 3qxy - x^2$ . Write the answer in descending powers of  $x$ , collecting coefficients of like powers in a bracket.

3. Divide  $x^5 - 2x^4 - 4x^3 + 19x^2 - 31x + 12 + a$  by  $x^2 - 7x + 5$ , and find the value of  $a$  which makes the dividend exactly divisible by the divisor.

4. Find the factors of  $a^4 + a^2x^2 + x^4$ , and of  $(5a - 7b - 3c)^2 - (2a + 4b - c)^2$ ; and show that  $(b-c)^3 + (c-a)^3 + (a-b)^3 = 3(a-b)(b-c)(c-a)$ .

5. Prove the rule for finding the lowest common multiple of two algebraical expressions.

Find the L.C.M. of  $6x^2y^2(x^3 - y^3)$ ,  $15x^2y^3(x^2 + xy + y^2)$ ,  $3x^2y(x - y)^2(x + y)$ ,  $4(x + y)^3(x - y)^2xy$ .

6. Express in terms of  $a$  and  $b$  in its simplest form the fraction

$$\frac{x-y}{x+y} + \frac{x+y}{x-y}, \text{ when } x = \frac{a-b}{a^2-ab+b^2} \text{ and } y = \frac{a+b}{a^2+ab+b^2}.$$

7. Simplify—

$$\frac{(1 - \frac{1}{3} - x) \{ \frac{1}{2} - (1 - x) \}}{x - \frac{1}{6} - \frac{1}{6(1-x)}};$$

$$\text{and divide } 2 - \frac{3n}{m} + \frac{9n^2 - 2m^2}{m^2 + 2mn} \text{ by } \frac{1}{m} - \frac{1}{m - 2n - \frac{4n^2}{m+n}}.$$