

5. Prove the formulæ—

(1.) $\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$.

(2.) $\sin 3A = 3 \sin A - 4 \sin^3 A$.

(3.) $2 + 4 \cot^2 2A = \tan^2 A + \cot^2 A$.

(4.) $(\cot 2A + \operatorname{cosec} 2A)(\sec A + 1) = \cot \frac{A}{2}$.

6. If $\sin \theta = \sin a \sin(\phi + \theta)$, find $\tan \theta$ in terms of a and ϕ .

7. Find by logs. the value of—

$$\frac{\frac{1}{6}\left(\frac{13}{14}\right)^{\frac{1}{3}} \times .044 \times (846)^{\frac{1}{7}}}{13 + (152)^{\frac{1}{2}} \div (34 + [.186]^{\frac{1}{4}})}$$

8. What is meant by solving a triangle? Investigate formulæ for solving a right-angled triangle.

9. Solve the right-angled triangle ABC, having given $A = 53^\circ 19' 37''$, $C = 90^\circ$, $AB = 342.79$.

10. Solve the triangle ABC when the parts given are, $B = 39^\circ 15'$, $C = 13^\circ$, $AC = 217.3$.

11. Wishing to ascertain the breadth of a river, I measured a base line (BC) of 200 yards along its bank, and at the ends of this line I measured the angles between it and an object A on the opposite bank, which I found to be—

$$\begin{aligned} \angle BCA &= 30^\circ 15', \\ \angle CBA &= 22^\circ 17'. \end{aligned}$$

What was the breadth of the river?

ALGEBRA.

1. From $(a-b)(a+b)(2a-b)$ take $a^2(a-b) \times b^2(a+b)$; and give value of the result when $a+b=7$ and $a-b=3$.

2. Multiply $x^{\frac{3}{2}} - xy^{\frac{1}{2}} + x^{\frac{1}{2}}y - y^{\frac{3}{2}}$ by $x + x^{\frac{1}{2}}y^{\frac{1}{2}} - y$.

3. Divide $a^{2n+1} - a^{n+1} - a^n + a^{n-1}$ by a^{n-1} .

4. Find the highest common factor of $a^3 - a^2 + a - 1$, $a^3 - a^2 - a + 1$, and $3a^2 - 2a - 1$.

5. Simplify $\frac{a + \frac{1 + \frac{a}{b}}{b}}{a - \frac{1 - \frac{a}{b}}{b}} (a^6 - b^6)$.

6. Express with root symbols and positive indices $\frac{x^{-2}}{y^{\frac{2}{3}}} + \frac{x^{-\frac{1}{2}}}{y^{-1}} + \frac{x^{-\frac{2}{3}}}{y^{-\frac{2}{3}}}$; and with fractional indices $3\sqrt{a^2} + (3\sqrt{a})^5 + a\sqrt{a^3}$.

7. Extract the square root of $\frac{1}{4}a^2 + a(c-3e) + c(c-6e) + 9e^2$.

8. Solve $\begin{cases} \frac{4x+3y}{9} + 1 - (y+3) + \frac{7x-3}{6} = 10. \\ 4y - \frac{1}{3}x + \frac{2x+3y+1}{7} - (x+y-2) = 3. \end{cases}$

9. Solve $\frac{x^2}{3} - \frac{x}{5} - 70 = 2$.

10. John, you, and I have £40 10s. amongst us. If from half John's and mine yours were taken we should be 30s. in debt; and your money and mine are in the proportion 29:2. How much have we each got?

11. If I go to my office by one route I save 120 yards; if to the post office by one route I save 440 yards. The shorter routes are in the ratio of 2 to 3, and the longer of 1 to 2. What are the distances?

GEOMETRY.

1. Define a point and a line. Is there anything wanting in these definitions, and, if so, what?

2. If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other.

3. To a given straight line apply a parallelogram which shall have an angle equal to a given rectilineal angle, and shall be equal to a given rectilineal figure.

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts.

5. If two circles touch each other externally in any point, the straight line which joins their centres shall pass through that point of contact.

6. Inscribe an equilateral and equiangular hexagon in a given circle.

CHEMISTRY.

1. Give the composition and properties of the various oxides of nitrogen.

2. How is nitric acid made? Give formulæ. What are its properties?

3. You have given you potassic carbonate and sulphuric acid. What chemical reactions can you obtain? Give formulæ.

4. How is hydrofluoric acid prepared, and for what is it used in the arts?

5. Describe the properties of ozone, and state in what respects it differs from oxygen, of which it is an allotropic form.